

Chiral symmetry in nuclei: partial restoration and its consequences

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Abstract. Partial restoration in nuclear matter of the chiral symmetry of QCD is discussed together with some of its possible signals. Estimates of corrections to the leading, linear dependence of the quark condensate are found to be small, implying a significant reduction of that condensate in matter. The importance of the pion cloud for the scalar quark density of a single nucleon indicates a close connection between chiral symmetry restoration and the attractive two-pion exchange interaction between nucleons. This force is sufficiently long-ranged that nucleons in nuclear matter will feel a significant degree of symmetry restoration despite the strong correlations between them. Expected consequences of this include reductions in hadron masses and decay constants. Various signals of these effects are discussed, in particular the enhancement of the axial charge of a nucleon in matter.

1. Introduction

Long before the advent of QCD, chiral symmetry was known to be an essential feature of the strong interaction [1]. Indeed it was this symmetry and its associated current algebra that led first to the invention of the quark model [2] and thence to QCD itself. Chiral symmetries appear in theories with massless fermions, where the fields describing right- and left-handed particles decouple. They are preserved in gauge theories by interactions with vector fields, at least in the absence of anomalies. In contrast, interactions with a Lorentz scalar character couple right- and left-handed fields and so break chiral symmetries.

Our theory of the strong interaction, QCD, possesses an approximate chiral symmetry because the up and down quarks have current masses, generated by their coupling to the electroweak Higgs field, that are very much smaller than the basic energy scale Λ_{QCD} . The same is true to a lesser extent for the strange quark. To the extent that we can neglect these masses QCD has separate isospin symmetries for the right- and left-handed quarks. Hence the symmetry group is referred to as $SU(2)_R \times SU(2)_L$. This can

be extended to three flavours although obviously the corresponding symmetry is more strongly broken.

The conservation of the currents associated with these symmetries controls the form of many of the interactions among pions and nucleons [1, 3]. Yet the symmetry is not obvious in the spectrum of hadrons – no massless fermions or parity doublets are seen. Instead the QCD vacuum is not invariant under chiral rotations and the symmetry is hidden or, to use the standard but somewhat misleading phrase, “spontaneously broken” [4]. The vacuum can be thought of as a condensed state of quark-antiquark pairs, with strong analogies to the condensate of Cooper pairs in a superconductor or the Higgs vacuum in electroweak theory. The order parameter that describes the hidden chiral symmetry of the strong interaction is the scalar density of quarks, often called the quark condensate.

There is an important difference between QCD and a superconductor or Higgs model: the chiral symmetry is global and its currents are not coupled to gauge fields. Hence vacua with different orientations of the order parameter are distinguishable. For an exact symmetry there would be no restoring force against chiral rotations of the vacuum and this would lead to the appearance of massless particles, known as “Goldstone bosons.” In QCD the chiral symmetry is explicitly broken by the current masses of the quarks and so the corresponding particles are not exactly massless. Nonetheless the pions have masses that are very much smaller than all other hadron masses, showing that they are close to being the Goldstone bosons of hidden chiral symmetry. The kaon masses are somewhat larger and so those particles are further from being approximate Goldstone bosons.

As with a superconductor we expect to return to a “normal” phase where the symmetry is restored, either at high temperatures or in strong external fields. The high-temperature phase of QCD is the quark-gluon plasma, which must have existed in the early universe and which may be recreated in ultra-relativistic heavy-ion collisions [5]. At zero temperature we expect symmetry restoration when the density of baryons becomes high enough. This might possibly occur in the cores of neutron stars, converting them to quark stars [6]. More importantly, precursors of that transition may already be present at ordinary nuclear densities. In that case the interior of a nucleus could be regarded as a laboratory where we can probe the physics of symmetry restoration in QCD [7, 8, 9].

Partial restoration of chiral symmetry inside nuclei could form an important part of the nuclear binding energy. By reducing the dynamical masses of the quarks, it could change the masses of nucleons and mesons in the medium and even their structures. Such modifications would alter the interactions of nuclei with electromagnetic and weak probes, and would contribute to the density dependence of nuclear forces.

The starting point for any discussion of chiral symmetry in nuclei is the expression for the leading density dependence of the the scalar density of quarks in nuclear matter [10],

$$\frac{\langle \bar{q}q \rangle_\rho}{\langle \bar{q}q \rangle_0} = 1 - \frac{\sigma_{\pi N}}{f_\pi^2 m_\pi^2} \rho. \quad (1.1)$$

This form is model-independent [11], but higher-order terms are not. It expresses the change from the vacuum quark condensate in terms of the baryon density ρ and the pion-nucleon sigma commutator $\sigma_{\pi N}$: a measure of the scalar density of quarks in the nucleon. Taking the recent value $\sigma_{\pi N} = 45 \pm 7$ MeV [12] suggests a $\sim 30\%$ reduction in

the condensate at nuclear matter densities and a phase transition at about three times normal nuclear densities. That could have dramatic consequences for the properties of nucleons and mesons in matter.

Estimates based on the leading density dependence (1.1) should not be taken too seriously until higher order effects have been calculated. Moreover (1.1) refers to the spatial average of the quark condensate. The strong short-range correlations between nucleons could mean that such an average is not a particularly useful quantity. There are three questions that need to be addressed:

- Are there significant corrections to the estimate (1.1) for the quark condensate inside nuclei?
- How do correlations between nucleons affect the degree of symmetry restoration?
- What are the consequences of partial symmetry restoration for nucleon and meson properties?

None of these has a definitive answer as yet; this review describes the current state of our understanding and indicates directions for further investigation.

Sec. 2 sets the scene by outlining the basic features of chiral symmetry and introducing various approaches being used to study its restoration in nuclei. The quark condensate (Sec. 2.1) and the sigma commutator (Sec. 2.2) are treated in some detail since, as can be seen from (1.1), they are central to questions of symmetry restoration in nuclei. The importance of the pion cloud surrounding a nucleon is stressed since this contributes a long-range component to the scalar quark density of a nucleon. This provides a connection between symmetry restoration and the attractive two-pion exchange interaction between nucleons (Sec. 2.6).

Two models often used in the discussion of chiral symmetry in nuclei are the linear sigma and the Nambu–Jona-Lasinio models. These are introduced in Secs. 2.3 and 2.4 respectively. The first of them is based on a scalar, isoscalar meson field which represents the quark condensate together with pion fields which provide the corresponding Goldstone bosons. This embodies all the basic features of hidden chiral symmetry and can be used to illustrate the qualitative aspects of symmetry restoration. The second attempts to provide a closer model for QCD by including only quarks, with interactions that dynamically generate a quark condensate and bind quarks to form mesons.

QCD sum rules have also become popular as a way to relate hadron properties to condensates, without introducing model assumptions. The basic features of this approach are described in Sec. 2.5, and illustrated by an application to the nucleon mass in vacuum. This shows that an important contribution to the nucleon mass arises from the quark condensate.

Changes of the quark condensate in nuclear matter are described in Sec. 3.1, starting with the model-independent form (1.1) for the linear density dependence. Higher-order corrections are examined, first in schematic treatments based on the linear sigma and NJL models, and then somewhat more realistically. Although the simple models can provide instructive qualitative pictures, their quantitative results should not be taken seriously. More realistic estimates of meson-exchange contributions from pions and heavier mesons suggest that corrections to (1.1) are small, at least at normal nuclear densities.

The importance of two-pion exchange, which has a fairly long range, means that correlations between the nucleons should not greatly reduce the effects of symmetry restoration. In addition it implies a connection between the changes in the quark condensate and the phenomenological scalar fields of relativistic nuclear phenomenology. This is also suggested by the application of QCD sum rules to the self-energy of a nucleon in matter (Sec. 3.4).

The likely effects of partial symmetry restoration on hadrons in matter (Secs. 3.2 and 3.3) include decreases in hadron masses and meson decay constants, as well as modifications of nucleon couplings and form factors. The masses of the approximate Goldstone bosons, pions and kaons, could behave rather differently. They are of particular interest because of recent suggestions that *s*-wave kaon condensation could occur in dense nuclear matter. Present estimates of their behaviour are discussed in Sec. 3.2. For the nucleon, a decrease in its mass is expected along with changes in the strengths and form factors for electromagnetic and weak interactions (Sec. 3.3). It has been suggested that decreases in hadron masses might arise from a universal scaling related to the scale anomaly of QCD, but in Sec. 3.5 such changes are shown to be much smaller than those driven directly by the quark condensate.

Possible observable consequences are described in Sec. 4. In many cases, such as electromagnetic interactions with nuclei, other more conventional mechanisms also contribute and current calculations are not sufficiently accurate to distinguish whether symmetry-restoration effects are also present. The important exception is the axial charge (Sec. 4.1), whose enhancement provides good evidence for strong scalar fields in nuclei. Other less conclusive signals surveyed in Sec. 4.2 include quasi-elastic electron scattering and elastic K^+ scattering. Suggestions that the Nolen-Schiffer anomaly, seen in the energy differences between mirror nuclei, might be a consequence of partial symmetry restoration are discussed in Sec. 4.3. Changes in nucleon structure or meson masses could also have important effects on nuclear forces (Sec. 4.4). Finally a brief summary is given in Sec. 5.

2. Chiral symmetry

In the limit of massless quarks, the QCD Lagrangian is invariant under both ordinary isospin rotations,

$$\psi \rightarrow (1 - \frac{1}{2}i\boldsymbol{\beta} \cdot \boldsymbol{\tau})\psi, \quad (2.1)$$

and axial isospin rotations,

$$\psi \rightarrow (1 - \frac{1}{2}i\boldsymbol{\alpha} \cdot \boldsymbol{\tau}\gamma_5)\psi, \quad (2.2)$$

where $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ denote infinitesimal parameters. By taking combinations of these rotations involving $1 \pm \gamma_5$ we can independently rotate the isospin of right- and left-handed massless quarks. Hence the symmetry is referred to as $SU(2)_R \times SU(2)_L$. (I concentrate here on the up and down quarks; the extension to three light flavours is straight-forward.)

Chiral symmetry is respected by interactions with vector fields (such as gluons and photons) since $\bar{\psi}\gamma_\mu\psi$ is invariant under axial rotations. The scalar and pseudoscalar densities of quarks are not invariant, transforming under (2.2) as

$$\bar{\psi}\psi \rightarrow \bar{\psi}\psi - \boldsymbol{\alpha} \cdot \bar{\psi}i\boldsymbol{\tau}\gamma_5\psi,$$

$$\bar{\psi}i\boldsymbol{\tau}\gamma_5\psi \rightarrow \bar{\psi}i\boldsymbol{\tau}\gamma_5\psi + \boldsymbol{\alpha}\bar{\psi}\psi. \quad (2.3)$$

Hence fermion mass terms or couplings to scalar fields break the symmetry.

The Noether currents corresponding to the transformations (2.1,2) are the (vector) isospin current

$$\mathbf{V}^\mu = \bar{\psi}\gamma^\mu\frac{1}{2}\boldsymbol{\tau}\psi, \quad (2.4)$$

and the axial current

$$\mathbf{A}^\mu = \bar{\psi}\gamma^\mu\gamma_5\frac{1}{2}\boldsymbol{\tau}\psi. \quad (2.5)$$

In the absence of current quark masses, these would both be conserved. With such masses, the divergences of the currents are

$$\partial_\mu V_i^\mu = \frac{1}{2}\Delta m\epsilon_{i3j}\bar{\psi}\tau_j\psi, \quad (2.6)$$

and

$$\partial_\mu A_i^\mu = \bar{m}i\bar{\psi}\gamma_5\tau_i\psi + \frac{1}{2}\Delta m\delta_{i3}i\bar{\psi}\psi, \quad (2.7)$$

where \bar{m} is the average of the current masses for the up and down quarks, and Δm is their difference.

These currents are coupled to photons and W bosons. Hence their matrix elements can be extracted from the electromagnetic and weak interactions of hadrons. For example, the weak decay of charged pions involves [3]

$$\langle 0|A_i^\mu(x)|\pi_j(q)\rangle = if_\pi q^\mu e^{-iq\cdot x}\delta_{ij}, \quad (2.8)$$

where the pion decay constant is $f_\pi = 92.5 \pm 0.2$ MeV [13, 14].

We can imagine a world in which the current masses are zero. Even in that world, the “chiral limit,” the QCD vacuum would not be invariant under axial isospin rotations since $SU(2)_R \times SU(2)_L$ is a hidden symmetry. The pions would then appear as massless Goldstone bosons. In that limit the Goldstone boson nature of the pions would allow one to determine the interactions of low-momentum pions purely from chiral symmetry. Another consequence of the lack of invariance of the vacuum is the non-zero matrix element (2.8) of the axial current between the vacuum and one-pion states.

The small size of the pion masses compared with those of all other hadrons indicates that the real world is not too far from the chiral limit. An essential idea in elucidating the consequences of approximate chiral symmetry for strong interactions is that of “partial conservation of the axial current” (PCAC). An introduction to this can be found in the lectures of Treiman [15] and clear recent discussion of it can be found in [16].

PCAC starts from the observation that, with explicit symmetry breaking, the divergence of Eq. (2.8) is

$$\langle 0|\partial_\mu A_i^\mu(x)|\pi_j(q)\rangle = f_\pi m_\pi^2 e^{-iq\cdot x}\delta_{ij}. \quad (2.9)$$

This shows that the operators

$$\phi(x) = \partial_\mu \mathbf{A}^\mu(x)/(f_\pi m_\pi^2) \quad (2.10)$$

connect the vacuum and one-pion states with the same normalisation that canonical pion fields would have. We can therefore use these operators as so-called “interpolating” pion fields. Of course this is a matter of choice: any operators that can connect these

states could be used as interpolating fields. Note that all such fields should give the same results for all physical amplitudes involving on-shell pions; where they differ is in their off-shell extrapolations. The advantage of the PCAC choice is that, by going to the soft-pion limit $q \rightarrow 0$, we can relate amplitudes involving pions to the axial transformation properties of the states.

The crucial dynamical assumption embodied in PCAC is that any matrix element of $\partial_\mu A_i^\mu$ has the form $(q^2 - m_\pi^2)^{-1}$ times a smoothly varying function of q^2 [16, 15]. By assuming that the variation of these functions over the range $q^2 = 0$ to m_π^2 is small, one can derive various low energy theorems. Corrections to these can be investigated systematically in increasing powers of the current quark masses, a technique known as chiral perturbation theory (ChPT) [3, 17, 18].

2.1. Quark condensate

Questions of partial symmetry restoration and its consequences focus on the changes in the quark condensate in nuclear matter. The starting point is obviously the quark condensate in the QCD vacuum. A value for this is found by applying PCAC to the vacuum expectation value of the time-ordered product of two interpolating pion fields:

$$\frac{1}{3} \sum_i \int d^4x e^{iq \cdot x} \langle 0 | T(\partial_\mu A_i^\mu(x), \partial_\nu A_i^\nu(0)) | 0 \rangle = i \frac{f_\pi^2 m_\pi^4}{q^2 - m_\pi^2} f(q^2), \quad (2.12)$$

where on the pion mass shell $f(m_\pi^2) = 1$. I have taken an isoscalar combination of fields here so that the result will not be sensitive to the difference between the quark masses.

The soft-pion limit of this amplitude is obtained by considering pions with zero three-momentum $\mathbf{q} = 0$, and then taking $q^0 \rightarrow 0$. Integrating by parts and taking this limit allows us to rewrite the l.h.s. of (2.12) in the form

$$\frac{1}{3} \sum_i \langle 0 | [Q_5^i, \partial_\nu A_i^\nu(0)] | 0 \rangle = i \frac{1}{3} \sum_i \langle 0 | [Q_5^i, [Q_5^i, \mathcal{H}(0)]] | 0 \rangle, \quad (2.13)$$

where the Q_5^i are the axial charge operators and $\mathcal{H}(x)$ is the Hamiltonian density. If we use the PCAC assumption that $f(0) \simeq f(m_\pi^2)$ then we find that the soft-pion limit of (2.12) gives

$$\frac{1}{3} \sum_i \langle 0 | [Q_5^i, [Q_5^i, \mathcal{H}(0)]] | 0 \rangle \simeq -f_\pi^2 m_\pi^2. \quad (2.14)$$

This double commutator picks out the (isoscalar) part of the Hamiltonian that breaks the symmetry and so (2.14) gives a connection between the pion mass and the strength of the symmetry breaking, known as a Gell-Mann–Oakes–Renner (GOR) relation [19]. The form of this equation shows that it is an energy-weighted sum rule; PCAC is equivalent to assuming that it is saturated by a single state, namely the pion.

The symmetry-breaking part of the QCD Hamiltonian is $\bar{m}\bar{\psi}\psi$. Hence the GOR relation takes the form

$$\bar{m} \langle 0 | \bar{\psi}\psi | 0 \rangle \simeq -f_\pi^2 m_\pi^2. \quad (2.15)$$

If we know the quark masses then we deduce a value for the quark condensate. The current masses of the light quarks have been estimated from hadron mass splittings and QCD sum rules [20]. Unfortunately neither of these methods is very precise and, in addition, both the masses and the quark condensate depend on the choice of renormalisation

scale. Typical values for \overline{m} lie in the range 5–10 MeV, for a scale of 1 GeV. Since \overline{m} is not fixed within a factor of two, the quark condensate (usually quoted per quark flavour) is similarly uncertain:

$$\langle \overline{q}q \rangle \equiv \frac{1}{2} \langle \overline{\psi}\psi \rangle \simeq -(210 \text{ MeV})^3 \quad \text{to} \quad -(260 \text{ MeV})^3. \quad (2.16)$$

The quark condensate in the QCD vacuum is negative; the positive scalar densities associated with the quarks and antiquarks present in hadronic matter always tend to cancel some of the condensate, pushing the vacuum towards symmetry restoration.

As noted by Cohen *et al* [11], the Feynman-Hellmann theorem provides a useful way to think about quark densities in terms of the dependence of energies on the current quark mass. If $|\Psi(\overline{m})\rangle$ is a normalised eigenstate of the QCD Hamiltonian with energy $E(\overline{m})$, then the variational principle leads to

$$\overline{m} \langle \Psi(\overline{m}) | \int d^3\mathbf{r} \overline{\psi}\psi | \Psi(\overline{m}) \rangle = \overline{m} \frac{dE}{d\overline{m}} \simeq m_\pi^2 \frac{dE}{dm_\pi^2}. \quad (2.17)$$

The second, approximate equality holds to leading order in \overline{m} and follows from the GOR relation. As an example, consider a zero-momentum pion state: this has energy m_π and so (2.17) gives $\overline{m} \langle \pi | \int d^3\mathbf{r} \overline{\psi}\psi | \pi \rangle \simeq \frac{1}{2} m_\pi$. This corresponds to a (volume-integrated) scalar density for the pion in region of 7–14. This surprisingly large number shows that the pion is a highly collective state, rather than a simple $q\overline{q}$ pair.

2.2. Sigma commutator

In studies of medium effects on the quark condensate, a crucial role is played by the scalar density of quarks inside a nucleon. By analogy with the quark condensate in previous section, we can define a quantity known as the pion-nucleon sigma commutator [21]:

$$\sigma_{\pi N} = \frac{1}{3} \sum_i \langle N | [Q_5^i, [Q_5^i, H]] | N \rangle, \quad (2.18)$$

where $|N\rangle$ denotes a zero-momentum nucleon state. The commutator is equal to

$$\sigma_{\pi N} = \overline{m} \langle N | \int d^3\mathbf{r} \overline{\psi}\psi | N \rangle. \quad (2.19)$$

Hence $\sigma_{\pi N}$ is both the contribution of chiral symmetry breaking to the nucleon mass and a measure of the scalar density of quarks in the nucleon.

PCAC allows us to relate $\sigma_{\pi N}$ to the soft-pion limit of πN scattering [21] and hence a value for it can be deduced by extrapolation from physical πN scattering amplitudes. The most recent determination gives $\sigma_{\pi N} = 45 \pm 7 \text{ MeV}$ [12], although it should be remembered that there are inconsistencies between the data sets used in the extrapolation [22]. This value is significantly smaller than earlier estimates [23], mainly as a result of a much softer form factor. Gasser *et al* [12] find a radius of about 1.3 fm for the scalar form factor of the nucleon and hence a roughly 15 MeV difference between $\sigma_{\pi N}$ at $q^2 = 0$ and the value at the Cheng-Dashen point, $q^2 = 2m_\pi^2$.

For $\overline{m} \simeq 5\text{--}10 \text{ MeV}$, the above value for $\sigma_{\pi N}$ suggests a scalar density of quarks in a nucleon of about $\langle N | \int d^3\mathbf{r} \overline{\psi}\psi | N \rangle \simeq 4\text{--}10$. This is at least twice what one would expect in simple quark models. In relativistic models, such as bag [25] or soliton models [24], the valence quark contribution is roughly 2, lower than the naive result of 3 because the

scalar density of relativistic fermions is reduced by a factor M/E compared to the usual (vector) density.

The enhancement of scalar density deduced from $\sigma_{\pi N}$ over that of the valence quarks indicates a significant contribution from quark-antiquark pairs in the nucleon. One can think of the valence quarks distorting the condensate around them, partially restoring chiral symmetry in their neighbourhood [26]. The size and range of this distortion are central to questions of symmetry restoration in nuclear matter. To estimate its extent, we need to know the restoring forces acting against symmetry restoration.

Two effects may play important roles in the partial restoration of chiral symmetry around a nucleon. One is a mean field of scalar, isoscalar mesons which can be thought of as a direct deformation of the condensate. The other is the pion cloud of the nucleon. These mechanisms are coupled through the strong mixing between scalar mesons and two-pion channels.

Many approaches, in particular those based on the linear sigma and NJL models, treat the meson fields at the classical or tree level, and so omit the pion cloud of the nucleon. They focus on the role of the scalar, isoscalar meson which is the chiral partner of the pion. This particle, usually denoted σ , is the excitation quantum of the quark condensate. At tree level the forces acting against symmetry restoration are proportional to the square of its mass. If this σ mass is large, then the strong restoring force leads to a very small tree-level sigma commutator, as illustrated by the linear sigma model result in Sec. 2.3. On the other hand, if the sigma mass is low, as in the NJL model, then the observed sigma commutator can be reproduced without invoking the meson cloud (Sec. 2.4).

Such low values for the σ mass, $M_\sigma \simeq 600$ MeV, are similar to those used for the phenomenological scalar fields in relativistic models of nuclei [27, 28]. This invites the identification of those fields with the change in the quark condensate in matter. However there is no evidence for such a light scalar in the meson spectrum [29]. The most likely candidate for the chiral scalar particle is the $f_0(1400)$ of the data tables [14] but, as Au, Morgan and Pennington [30] have pointed out, this resonance may be better thought of as a broad structure in $\pi\pi$ scattering at around 1 GeV. The phenomenological scalar fields with much lower masses should rather be regarded as modelling correlated two-pion exchange, as discussed in Sec. 2.6.

A large mass of 1 GeV or more for the chiral σ field suggests a strong force acting against chiral symmetry restoration. If the coupling to pions were neglected then any changes in the quark condensate would be small and very short-ranged. In any chirally symmetric model of nucleon structure, the pion cloud contributes significantly to many observables, and in particular to $\sigma_{\pi N}$, because of the large scalar density in the pion mentioned in the previous section. These contributions are long-ranged because the pions are light. Calculations of $\sigma_{\pi N}$ in cloudy bag [31] and nontopological soliton models [32] (see also the linear sigma model calculation discussed in Sec. 2.6) find an extra 20–25 MeV from the cloud which, combined with the valence part, gives agreement with the observed value. Empirical support for a substantial pion-cloud contribution comes from the large radius for the scalar density of the nucleon found in recent analyses [12, 33].

Calculations of $\sigma_{\pi N}$ from first principles using lattice QCD are still at a preliminary stage. Older calculations, using the quenched approximation and with rather large current masses, found values similar to the valence contribution only [34]. More recent

results for dynamical fermions show significant disconnected contributions to $\sigma_{\pi N}$ [35] which suggest that meson loop effects are important even at the current quark masses of about 35 MeV that can be reached in present lattice calculations. Note that the pion cloud contains pieces corresponding to connected diagrams (the only ones present in the quenched approximation) as well as disconnected ones [36, 32]. Hence one cannot simply interpret these diagrams as valence and sea contributions respectively.

Another indicator of the importance of the cloud is the long-standing discrepancy between estimates of $\sigma_{\pi N}$ from the spectrum of octet baryons and those from πN scattering. If one assumes that the octet splittings are given by first-order perturbation theory in the current masses and that there are no strange quarks in the proton, then the σ commutator is [37]

$$\sigma_0 = \frac{\overline{m}}{m_s - \overline{m}} (M_\Xi + M_\Sigma - 2M_N). \quad (2.20)$$

More generally, one can allow for a nonzero density of strange quarks and write

$$\sigma_{\pi N} = \frac{\sigma_0}{1 - y}, \quad (2.21)$$

where

$$y = \frac{\langle N | \overline{s}s | N \rangle}{\langle N | \overline{u}u + \overline{d}d | N \rangle}. \quad (2.22)$$

If one takes $m_s/\overline{m} = 25$ [20], then the observed baryon masses lead to $\sigma_0 \simeq 25$ MeV. Naively this would suggest a large strange-quark content in the proton, $y \sim 0.5$, and hence a huge contribution to the nucleon mass from strange quarks [38]. Before taking such a conclusion seriously, one should examine whether first-order perturbation theory is valid for the baryon masses. As first pointed out by Jaffe [39] in a chiral bag model and subsequently shown in other models with strong meson clouds [40], there are strong nonlinearities in the dependence on the current masses. Kaon-cloud effects on baryon observables are very much smaller than those of pions. Hence the estimate (2.20), which is dominated by the strange-quark density of the hyperons, gives essentially the valence-quark piece of $\sigma_{\pi N}$ only.

Gasser [41, 20] has estimated corrections to (2.20) using ChPT. Non-analytic dependences of baryon masses on \overline{m} (or equivalently m_π^2) arise from the longest-range parts of the pion cloud. Chiral symmetry requires that logarithmic terms appear only at the order $m_\pi^4 \ln m_\pi$. The leading non-analytic term in $\sigma_{\pi N}$ is of order m_π^3 :

$$\sigma_{\pi N} = C m_\pi^2 - \frac{9}{64\pi^2} \frac{m_\pi^3}{f_\pi^2} + \dots \quad (2.23)$$

Gasser's results indicate that such terms can raise the estimate to $\sigma_0 \sim 35$ MeV. Moreover that calculation includes πN loops only. Virtual $\Delta\pi$ states have long been known to give significant contributions to nucleon properties in the cloudy bag [25], and recently Jenkins and Manohar have pointed out their importance in ChPT [42] (see also: [43, 44, 45]). In the case of $\sigma_{\pi N}$, bag and soliton models give $\Delta\pi$ contributions of 6–10 MeV [31, 32]. These can remove the remaining discrepancy between the two estimates of $\sigma_{\pi N}$ without requiring any substantial strange-quark content in the nucleon.

A similar kaon-nucleon sigma commutator can be defined as

$$\sigma_{KN} = \frac{1}{2}(m_u + m_s) \langle N | \int d^3\mathbf{r} (\overline{u}u + \overline{s}s) | N \rangle, \quad (2.24)$$

where an average over proton and neutron is understood. In principle this could be extracted from KN scattering, but the much larger extrapolations involved mean that σ_{KN} is very poorly determined [46]. If the strange-quark content in the proton is small, σ_{KN} is to a good approximation just $m_s/4\bar{m}$ times $\sigma_{\pi N}$. This estimate suggests that σ_{KN} is at least ~ 280 MeV.

2.3. Linear sigma model

Although lattice QCD can give us essential information about the chiral phase transition at high temperatures [47], high density calculations are still in a very preliminary state [48] since the Monte-Carlo integration methods have difficulty coping with chemical potentials. We are therefore forced to use QCD-motivated models to study that regime. The simplest model for the physics of symmetry restoration is the linear sigma model [49], which has a venerable history of applications to the hidden chiral symmetry of the strong interaction.

The model is based on a scalar, isoscalar field that represents the quark condensate, together with pseudoscalar, isovector pion fields. These transform like the corresponding quark bilinears (2.3) under axial rotations:

$$\sigma \rightarrow \sigma - \boldsymbol{\alpha} \cdot \boldsymbol{\phi}, \quad \boldsymbol{\phi} \rightarrow \boldsymbol{\phi} + \boldsymbol{\alpha} \sigma. \quad (2.25)$$

These fields can be coupled in a chirally invariant way to an isospin doublet of fermions. The fermions can be either nucleons, if we are interested in nuclear physics, or quarks, if we want to describe baryon structure [24]. Here I use the original version involving nucleon fields [49]. In either case the model Lagrangian takes the form

$$\mathcal{L} = \bar{\psi}[i\not{\partial} + g(\sigma + i\boldsymbol{\phi} \cdot \boldsymbol{\tau}\gamma_5)]\psi + \frac{1}{2}(\partial_\mu\sigma)^2 + \frac{1}{2}(\partial_\mu\boldsymbol{\phi})^2 - U(\sigma, \boldsymbol{\phi}). \quad (2.26)$$

The symmetry is hidden if the potential U is chosen to have a “Mexican-hat” form:

$$U_0(\sigma, \boldsymbol{\phi}) = \frac{\lambda^2}{4}(\sigma^2 + \boldsymbol{\phi}^2 - \nu^2)^2. \quad (2.27)$$

As can be seen from Fig. 2.1, there is a circle of degenerate minima in the brim of the hat. If we take the physical vacuum to have good parity ($\sigma = \pm\nu$, $\boldsymbol{\phi} = 0$) there is no restoring force against pionic excitations about the vacuum. The pions are thus massless Goldstone bosons. The σ field experiences a strong restoring force and so its excitations are massive scalar mesons.

We can give the pions their observed masses by tipping the Mexican hat so that the symmetry is broken and the vacuum is unique. The full potential is then

$$U(\sigma, \boldsymbol{\phi}) = \frac{\lambda^2}{4}(\sigma^2 + \boldsymbol{\phi}^2 - \nu^2)^2 + f_\pi m_\pi^2 \sigma. \quad (2.28)$$

With this choice of symmetry-breaking term, the model explicitly embodies PCAC: the divergence of the axial current is proportional to the pion field of the Lagrangian.

The matrix element for pion decay in the model fixes the vacuum expectation value of the σ field to be $-f_\pi$. The nucleon mass is thus

$$M_N = g f_\pi, \quad (2.29)$$

where g is the πN coupling constant. This is just the Goldberger–Treiman relation with $g_A = 1$. In terms of the parameters of the potential, the meson masses are

$$m_\sigma^2 = \lambda^2(3f_\pi^2 - \nu^2), \quad m_\pi^2 = \lambda^2(f_\pi^2 - \nu^2). \quad (2.30)$$

To illustrate of some of the features of this model, I revisit the calculation of soft-pion scattering from a nucleon in this model. The results of this will be used in Sec. 3. The diagrams that contribute at tree-level are shown in Fig. 2.2. Consider, for simplicity, scattering of a virtual pion of zero three-momentum but with energy ω from a nucleon at rest. Using (2.29, 30), the scattering amplitude can be written

$$T = \frac{M_N}{f_\pi^2} \left[\frac{m_\sigma^2 - m_\pi^2}{m_\sigma^2} - \frac{4M_N^2}{4M_N^2 - \omega^2} \right]. \quad (2.31)$$

Chiral symmetry ensures that terms of order m_π^0 (and ω^0) cancel. For soft pions, with $\omega = 0$, this amplitude takes the form required by PCAC [1, 21]:

$$T = -\frac{\sigma_{\pi N}}{f_\pi^2}, \quad (2.32)$$

where, at tree-level, the sigma commutator is

$$\sigma_{\pi N} = \frac{m_\pi^2}{m_\sigma^2} M_N. \quad (2.33)$$

This result (2.33) for $\sigma_{\pi N}$ can of course also be obtained directly as the matrix element of the symmetry-breaking term $f_\pi m_\pi^2 \sigma$. For a typical σ mass of 1200 MeV, it gives $\sigma_{\pi N} \simeq 13$ MeV. This is similar to the valence-quark contributions discussed in the previous section, and it indicates the need to go beyond tree-level by including loop diagrams corresponding to the pion cloud.

In the soft-pion limit, the contributions of diagrams 2.2(b) and (c) come from negative-energy intermediate states. In time-ordered perturbation theory such diagrams can be interpreted in terms of virtual nucleon-antinucleon states and hence the cancellation between these Z-graphs and σ exchange is often referred to as “pair suppression.”

Both scalar fields and Z-graphs play important roles in relativistic treatments of nuclei [27, 28, 50, 51]. It should be remembered that they describe short-distance physics which is not determined purely by chiral symmetry and which is thus model-dependent. The σ field is needed if one wants to describe chiral symmetry restoration using an effective field theory of nucleons and mesons. The Z-graphs are essential if current conservation and the associated low-energy theorems are to be maintained. Of course nucleons are not point-like Dirac particles and so it is likely that the interpretation of these diagrams as pair creation should not be taken too literally. Brodsky [52] has long argued that form factors should suppress pair creation of composite objects. Instead the Z-graphs should be regarded as mocking up the effects of nucleon structure [53]. Calculations in a nontopological soliton model [54] have shown that a combination of quark Z-graphs and excitations leads to the same results for several low-energy theorems.

The scattering amplitude (2.32) for soft pions is repulsive. From (2.22) one can see that at tree-level this repulsion increases with the pion energy. This increase arises from the energy denominators of the Z-graphs. In contrast the amplitude at the physical pion threshold (zero pion three-momentum and $\omega = m_\pi$) is very small [22]. Such a reduction can be obtained by including positive-energy intermediate states: pion loops or excited baryons are needed for a realistic description of πN scattering.

2.4. Nambu–Jona-Lasinio model

The sigma model can give us a good qualitative picture of the physics involved, but in QCD we have no fundamental scalar fields; really the σ and pion fields should be regarded as approximate descriptions of the corresponding bilinear combinations of quark fields. A model that is often proposed as a step closer to QCD is the remarkable one introduced by Nambu and Jona-Lasinio [55] in which chiral symmetry is hidden dynamically. It consists of fermions interacting via a local four-fermion interaction (a zero-range two-body force):

$$\mathcal{L}_{\text{NJL}} = \bar{\psi}(i\partial - \bar{m})\psi + \frac{G}{2} [(\bar{\psi}\psi)^2 + (\bar{\psi}\mathbf{i}\boldsymbol{\tau}\gamma_5\psi)^2]. \quad (2.34)$$

In the modern versions of this the fermions are interpreted as quarks [56]. The combination of scalar and pseudoscalar interactions is chosen since it is chirally symmetric. Obviously such a zero-range force is a caricature of the strong gluon-exchange interactions between quarks. Nonetheless it retains some of their important features, with the notable exception of confinement. Since it is non-renormalisable, the model only makes sense with a cut-off at short distances. A wide variety of cut-off procedures has been used [57]; fortunately the qualitative results do not depend on this choice. The model has also been extended to include strange quarks as well as vector and axial-vector interactions between the quarks [56].

In the one-loop approximation (a Hartree treatment of the Dirac sea), the dynamical quark mass satisfies the nonlinear equation,

$$M_q = \bar{m} + 4N_c N_f G \int^\Lambda \frac{d^4k}{(2\pi)^4} \frac{M_q}{k^2 + M_q^2}, \quad (2.35)$$

for N_c colours and N_f flavours of light quarks. I have given the form for a simple covariant cut-off, $k \leq \Lambda$, on the magnitude of the four-momentum (which has been Wick-rotated to Euclidean space-time). This equation always has the obvious solution $M_q = 0$, corresponding to a vacuum with manifest chiral symmetry. For values G greater than a critical $G_c(\Lambda)$, which depends on the cut-off, the lowest-energy solution gives a vacuum in which the quarks have a non-zero mass.

In the latter vacuum, the chiral symmetry is hidden. There is a quark condensate,

$$\langle \bar{q}q \rangle \equiv \frac{1}{N_f} \langle \bar{\psi}\psi \rangle = -4N_c \int^\Lambda \frac{d^4k}{(2\pi)^4} \frac{M_q}{k^2 + M_q^2}, \quad (2.36)$$

whose coupling to the quarks produces their mass (2.35), analogously to the vacuum expectation value of σ in the linear sigma model. The vacuum of the NJL model is a condensate of quark-antiquark pairs, and the energy difference $2M_q$ between the top of the Dirac sea and the lowest valence quark level can be thought of as the gap energy required to break a pair and form a quasiparticle-hole excitation.

The local nature of the interaction in this model means that the Bethe-Salpeter equation for quark-antiquark scattering has a simple form and can be solved by summing a geometric series. The bound-state poles of this amplitude can be used to determine meson masses and wave-functions. With no current quark masses the pions are massless Goldstone bosons, and the σ meson is an almost unbound state in the scalar, isoscalar channel, with a mass

$$m_\sigma \simeq 2M_q. \quad (2.37)$$

Current quark mass terms can be added to the Lagrangian (2.34), breaking the symmetry and giving the pions masses.

The model parameters are chosen to fit the pion decay constant and mass and give a dynamical quark mass in the range 300–400 MeV. The corresponding values for the cut-off are about 600–800 MeV, depending on the form used. These give reasonable values for the quark condensate, between $-(220 \text{ MeV})^3$ and $-(290 \text{ MeV})^3$.

Baryons can be constructed as solitons in this model [58]. This is generally done by bosonising the model, converting it into an equivalent model involving only meson fields. Auxiliary σ and pion fields are introduced to express the Lagrangian in a form that is bilinear in the quark fields,

$$\mathcal{L}'_{\text{NJL}} = \bar{\psi}[i\not{\partial} + g(\sigma + i\boldsymbol{\phi} \cdot \boldsymbol{\tau}\gamma_5)]\psi - \frac{1}{2}\mu^2(\sigma^2 + \boldsymbol{\phi}^2). \quad (2.38)$$

Integrating out the quark fields then leaves a purely bosonic effective action,

$$S_{\text{NJL}} = -i\text{Tr} \ln[i\not{\partial} + g(\sigma + i\boldsymbol{\phi} \cdot \boldsymbol{\tau}\gamma_5)] - \frac{1}{2}\mu^2 \int d^4x (\sigma^2 + \boldsymbol{\phi}^2). \quad (2.39)$$

The first term is the logarithm of the determinant of the Dirac operator. It is a complicated object whose dependence on the boson fields is highly nonlocal because of the effects of vacuum polarisation. Techniques have been developed for evaluating this determinant for localised soliton configurations and then minimising the effective action [58]. For uniform systems things are much simpler: the nonlocal terms do not contribute and effective action reduces to an effective potential. If the coupling strength is greater than the critical value, this potential has a similar form to the Mexican hat of the linear sigma model (although it is not simply a quartic function of the fields).

An obvious shortcoming of this model is that it does not absolutely confine quarks. Hence, for example, mesons with masses greater than $2M_q$ are not bound. Another problem is the lightness of the σ meson, typically around 700 MeV. Although it may be tempting to identify this with the light σ of nuclear physics, that should be avoided for the reasons discussed in Secs. 2.2 and 2.6. The low σ mass means that the NJL model underestimates the forces reacting against symmetry restoration. This results in large vacuum polarisation effects, even without inclusion of the pion cloud. For example, the pion-quark sigma commutator is about twice the naive expectation:

$$\sigma_{\pi q} = \bar{m} \frac{dM_q}{d\bar{m}} \simeq 2\bar{m}. \quad (2.40)$$

Also, the dressed quark has a scalar form factor whose radius is determined by the σ mass and is therefore large [59].

The softness of the vacuum in the NJL model can be removed by adding extra interactions. For example, Ripka and Jaminon [61] have suggested a variant in which the quadratic term in the “half-bosonised” Lagrangian (2.38) is replaced by a quartic. The model was originally motivated by ideas of scale invariance, but in fact the extra dilaton field plays very little role in the dynamics see Sec. 3.4 below). The quartic term could be thought of as arising from a four-body interaction among the quarks. It is equivalent to adding an extra Mexican-hat term to the effective potential. This has the effect of increasing the σ mass to $\sim 1.5 \text{ GeV}$ and correspondingly reducing the vacuum polarisation in this channel.

2.5. QCD sum rules

A rather different approach from the models just described is provided by the QCD sum rules developed by Shifman, Vainshtein and Zakharov [62]. These attempt to relate hadron properties to condensates: vacuum expectation values that describe the non-perturbative aspects of the QCD vacuum. Recently these sum rules have become popular as a possible tool for studying the behaviour of hadrons in nuclear matter.

The GOR relation of Sec. 2.1 can be thought of as a prototype for these sum rules: it uses a Green's function of interpolating fields to relate pion properties and the quark condensate. It is particularly simple because chiral symmetry ensures that only one state (the pion) dominates the propagator at low q^2 and that only one condensate appears. For interpolating fields corresponding to other mesons or baryons, such simplifications do not occur.

Consider a general Green's function of the form

$$\Pi(q) = i \int d^4x e^{iq \cdot x} \langle 0 | T(\eta(x), \bar{\eta}(x)) | 0 \rangle, \quad (2.41)$$

where $\eta(x)$ denotes an interpolating field with the quantum numbers of the hadron of interest. By inserting a complete set of states, one can express this in the form of a dispersion relation involving the spectral density of states in the chosen channel. This density can then be written in terms of the masses and couplings of the hadrons. Alternatively the operator-product expansion (OPE) [63] can be used to express the Green's function as a sum of vacuum matrix elements of local operators. These operators are combinations of quark and gluon fields whose matrix elements are the condensates representing the nonperturbative physics. Each is multiplied by a function of q^2 which can be calculated from perturbative QCD. By matching the two expressions for the same propagator, values for the condensates can be deduced from observed hadron properties.

The OPE is valid for large space-like momenta ($q^2 < 0$), but in that region many resonances contribute to the spectral representation of the propagator. A direct comparison of the two forms as functions of q^2 is not practical; instead a weighted integral is used [64, 65]. The art of the sum rule approach is to pick a weighting function that both emphasises the role of low-lying resonances in the spectral representation and keeps down the number of condensates making important contributions to the OPE. The choice of Shifman *et al* [62], which has been found to be particularly convenient, is the Borel transform. Although this can be expressed as a contour integral [66], it is normally written as

$$\mathcal{B}f(Q^2) = \lim_{Q^2, n \rightarrow \infty} \left(-\frac{d}{dQ^2} \right)^n f(Q^2) \equiv \hat{f}(M^2), \quad (2.42)$$

where $Q^2 = -q^2$ and n are taken to infinity while their ratio

$$M^2 = \frac{Q^2}{n}, \quad (2.43)$$

is kept constant. The result obviously depends on an arbitrary parameter M , the ‘‘Borel mass.’’

This transform eliminates any polynomials in Q^2 which arise from subtractions in the dispersion relation. More importantly, it exponentially suppresses contributions from high-mass states in the spectral representation. In the OPE, high dimension condensates

are suppressed by inverse powers of M^2 . While, on one hand one would like M to be small enough that the lowest state dominates in the spectral representation, one would also like M to be large to keep the number of condensates in the OPE manageable. The exponential suppression of higher resonances suggests that one may be able to find an intermediate range for M where these conflicting desires can be reasonably well satisfied. The fact the results should be independent of M provides a consistency check: if there is a region where the two representations agree and are flat functions of M , then one may have some confidence in the deduced values for the condensates.

To illustrate the sum rule concept, I outline here their application to the nucleon. Detailed discussions can be found in the review [64] and in Ref. [66]. There are two linearly independent ways in which an interpolating nucleon field can be constructed from one down- and two up-quark fields coupled to spin- $\frac{1}{2}$ and isospin- $\frac{1}{2}$. As with the Borel mass, one tries to find a field that both ensures that the nucleon dominates in the spectral representation and minimises the higher-order contributions in the OPE. The optimal choice is the one introduced by Ioffe [67] which can conveniently be expressed in a form where the up quarks are coupled to form a vector diquark:

$$\eta(x) = \epsilon_{abc}[u_a^T(x)C\gamma_\mu u_b(x)]\gamma_5\gamma^\mu d_c(x), \quad (2.44)$$

where a, b, c label the colours of the quark fields and C is the charge conjugation matrix. From Lorentz covariance, parity and time-reversal symmetries, the propagator (2.41) constructed with this field must have the form

$$\Pi(q) = \Pi_s(q^2) + \not{q}\Pi_q(q^2). \quad (2.45)$$

Using a dispersion relation, each of the scalar functions $\Pi_i(q^2)$ can be written as a nucleon pole plus continuum:

$$\Pi_s(q^2) = \frac{\lambda_N^2 M_N}{q^2 - M_N^2} - \int_{s_0}^{\infty} ds \frac{\rho_s^{\text{cont}}(s)}{q^2 - s} + \dots, \quad (2.46a)$$

$$\Pi_q(q^2) = \frac{\lambda_N^2}{q^2 - M_N^2} - \int_{s_0}^{\infty} ds \frac{\rho_q^{\text{cont}}(s)}{q^2 - s} + \dots, \quad (2.46b)$$

where the dots denote polynomials from subtractions. Alternatively the OPE yields the following for each of the terms

$$\Pi_s(q^2) = \frac{1}{4\pi^2} q^2 \ln(-q^2) \langle \bar{q}q \rangle + \dots, \quad (2.47a)$$

$$\Pi_q(q^2) = -\frac{1}{64\pi^2} q^4 \ln(-q^2) - \frac{1}{32\pi^2} \ln(-q^2) \langle \frac{\alpha_s}{\pi} G_{\mu\nu} G^{\mu\nu} \rangle + \dots. \quad (2.47b)$$

where I have shown explicitly only the contributions from the quark and gluon condensates with dimension less than five. Higher-dimensioned condensates appear multiplied by inverse powers of q^2 . Their full forms can be found in [68, 66].

The pole term in the spectral representation introduces an unknown strength λ_N for the coupling of the interpolating field to the nucleon. The continuum in the spectral representation (2.46) is often approximated by the perturbative continuum, arising from the cuts in the OPE expressions, starting at $q^2 = s_0$ which is treated as an adjustable parameter. Obviously this is not a particularly accurate representation, but provided

the Borel mass is not too large the weighted average in the sum rule is not very sensitive to the details of the continuum.

Equating the two expressions (2.46, 47) for each of the terms in the propagator and Borel transforming leads to two sum rules:

$$\lambda_N^2 M_N e^{-M_N^2/M^2} + \int_{s_0}^{\infty} ds e^{-s/M^2} \rho_s^{\text{cont}}(s) = -\frac{M^4}{4\pi^2} \langle \bar{q}q \rangle + \dots, \quad (2.48a)$$

$$\lambda_N^2 e^{-M_N^2/M^2} + \int_{s_0}^{\infty} ds e^{-s/M^2} \rho_q^{\text{cont}}(s) = \frac{M^6}{32\pi^4} + \frac{M^2}{32\pi^2} \langle \frac{\alpha_s}{\pi} G_{\mu\nu} G^{\mu\nu} \rangle + \dots. \quad (2.48b)$$

The continuum contribution can be taken over to the r.h.s. of each equation, where it modifies the coefficients of the lowest condensates. The renormalisation group can be used to improve these sum rules by summing up the leading logarithms of Q^2 in the coefficients of the condensates.

Ioffe's sum rule [67] for the nucleon mass is given by the ratio of these two sum rules in which the coupling to the interpolating field cancels. An oversimplified but instructive version of this [64] is obtained by dropping all but the quark condensate in the OPE and neglecting the continuum. This leaves an expression for the nucleon mass that is proportional to the quark condensate:

$$M_N = -\frac{8\pi^2}{M^2} \langle \bar{q}q \rangle. \quad (2.49)$$

Taking a typical value of 1 GeV for the Borel mass and the quark condensate from (2.16), one finds a nucleon mass in the region of 900 MeV. This shows that the quark condensate makes a major contribution to the nucleon mass, in agreement with expectations based on the chiral models of the previous sections. Since the sum rule (2.49) obviously depends strongly on M , higher condensates are essential if it is to yield quantitative results.

In fact there are strong cancellations among the terms neglected in (2.49), which is why it gives a reasonable estimate of the nucleon mass. The most important piece omitted from it is the four-quark condensate, $\langle (\bar{q}q)^2 \rangle$, which will be needed for the discussion of sum rules in matter. Like other four-quark and higher condensates, this is often estimated using the factorised or “vacuum dominance” ansatz [62],

$$\langle (\bar{q}q)^2 \rangle = \langle \bar{q}q \rangle^2. \quad (2.50)$$

However, even in the vacuum, there are indications that this assumption is violated by about a factor of two [65]. We shall see that the uncertainties associated with this condensate place severe limitations on the predictive power of QCD sum rules for hadrons in matter.

One should also remember that QCD sum rules focus on short-distance physics, as described by a truncated OPE. Long-range physics is subsumed into a simple parametrisation of the spectral function and so is not treated reliably. As pointed out by Griegel and Cohen [69], the simple ansatz for the continuum described above means that pion-cloud contribution to the nucleon mass is not properly described. In particular, the nonanalytic dependence of the nucleon mass on the current quark masses [41, 20] is not reproduced. The resulting uncertainty of ~ 100 MeV may not be too serious for the nucleon mass, but it does mean that the sigma commutator cannot be reliably estimated from QCD sum rules.

2.6. *NN interaction*

As alluded to in previous sections, scalar fields play a central role in relativistic nuclear physics. These are scalar, isoscalar fields, usually denoted by σ , that provide the attractive forces between nucleons in relativistic models of nuclear structure [27, 28] or nucleon scattering from nuclei [50, 51]. Their similarity to the fields used to model the quark condensate invites the question of whether such fields are related to partial restoration of chiral symmetry. Such a relationship has been suggested in the context of QCD sum rules [66], the linear sigma model [70] and the Nambu–Jona-Lasinio (NJL) model [71].

Phenomenological NN potentials [72, 74, 73] also find substantial intermediate range attraction in the scalar isoscalar channel. This attraction has long been known to be well described by exchange of two correlated pions [72, 75, 76]. The light σ particle of relativistic phenomenologies should be regarded as modelling this two-pion exchange process, and not an indication of a light chiral partner for the pion. Furthermore models with a light chiral partner for the pion give rise to strong many-body forces between nucleons [77] and cannot provide an acceptable description of nuclear properties [78].

Although the scalar fields of relativistic nuclear models are introduced to describe very different physics from the chiral partner of the pion, there is strong mixing of the chiral scalar with two-pion states, and this sharp distinction is lost. To explore the relationship between the attractive NN force and chiral symmetry restoration, I look at the calculation of two-pion exchange in the linear sigma model [76, 79, 80]. Other relevant work on chiral symmetry and the two-pion exchange interaction can be found in Ref. [81].

The interaction of interest can be found from the scalar, isoscalar piece of the irreducible scattering amplitude for two nucleons. The simplest contribution to this is just direct σ -exchange, Fig. 2.3. At one-loop order there are four diagrams involving exchange of a pair of virtual pions between the nucleons [80]. These are shown in Fig. 2.4. Working to this order, I have not included interactions between the exchanged pions. Such interactions are known to be essential to the strong attraction between the nucleons [72, 75] and hence this calculation cannot yield a realistic result for the full scalar interaction.

Direct σ exchange, Fig. 2.3, is purely scalar and isoscalar. Its contribution to the NN T -matrix is easily evaluated giving

$$S_D = - \left(\frac{M_N}{f_\pi} \right)^2 \frac{1}{m_\sigma^2 - t}. \quad (2.51)$$

The evaluation of the loop diagrams is long and tedious; details can be found in Ref. [79]. The scalar, isoscalar pieces of the T -matrix corresponding to Figs. 2.4(a)-(d) are denoted here by S_L , S_V , S_X and S_B respectively.

The amplitude for the box diagram, Fig. 2.4(d), arises from iterating one-pion exchange in the Bethe-Salpeter equation. To get an irreducible amplitude, the iterated one-pion exchange must be removed. However one cannot simply drop the whole contribution of 2.4(d) since, with pseudoscalar (PS) πN coupling; that would leave an irreducible amplitude that would not satisfy “pair suppression” (the constraints imposed by chiral symmetry mentioned in Sec. 2.3). That would not matter if the scattering equation were treated exactly, but any approximation would produce large violations of chiral symmetry. To avoid such problems phenomenological treatments of the NN

interaction normally use pseudovector (PV) πN coupling [82, 83, 73]. Subtracting the box diagram calculated with PV coupling leaves an irreducible scalar amplitude that can be compared with such interactions [76].

The full irreducible amplitude can be written

$$S = S_D + S_L + 2S_V + S_X + S_B - S_{B(PV)}, \quad (2.52)$$

where the detailed forms of the individual amplitudes can be found in Ref. [80]. Although the contributions from individual diagrams are large, there are strong cancellations between them as required by chiral symmetry. The net strength is about half that of a typical phenomenological σ exchange amplitude. This just shows that shows that interactions between the exchanged pions ought to be included, together with diagrams involving intermediate Δ 's [75, 73].

Of most interest for the present discussion is the piece involving direct σ coupling to one of the nucleons. These describe the degree of symmetry restoration experienced by that nucleon and is given by the sum of three diagrams:

$$S_{CR} = S_D + S_L + S_V. \quad (2.53)$$

The strength of this at zero momentum transfer is related to the sigma commutator, since in this model $\sigma_{\pi N}$ is just proportional to the matrix element of the σ field in a nucleon. Specifically one has

$$S_{CR}(t=0) = -\frac{M_N}{f_\pi^2 m_\pi^2} \sigma_{\pi N}. \quad (2.54)$$

The contribution from the pionic diagrams correspond to the cloud contributions to $\sigma_{\pi N}$ calculated in chiral bag and soliton models [31, 32], and for a cut-off of $\Lambda \simeq 1$ GeV they have a similar magnitude. They more than double $\sigma_{\pi N}$ compared with the tree-level result (2.33) which on its own gives S_D .

The form of this result (2.54) shows that it is much more general than the model studied here. It follows from the assumption that the nucleon mass is proportional to the quark condensate, and the fact that $\sigma_{\pi N}$ is a measure of the scalar quark density in the nucleon. Using the observed value of $\sigma_{\pi N}$ [12] gives $S_{CR} = -250 \text{ GeV}^{-2}$, which is comparable in strength to phenomenological scalar forces. Unlike the total scalar potential, this strength will not be changed by including $\pi\pi$ interactions, provided the couplings and cut-offs are chosen to reproduce the observed $\sigma_{\pi N}$. A crude estimate of the symmetry restoring potential in matter is ρS_{CR} , which is about -330 MeV at nuclear matter density, almost as large as phenomenological scalar potentials [27, 28, 50, 51]. The implications of this for a nucleon in matter will be discussed in Secs. 3.3 and 3.4.

3. Finite density

3.1. Quark condensate

The vacuum quark condensate is negative (2.15) while the scalar densities of quarks in hadrons are positive. As a result the net quark condensate will be smaller in nuclear

matter than in vacuum. At low densities we can treat the nucleons as independent and simply add their contributions to get the spatially averaged scalar density of quarks:

$$2\overline{m}\langle\overline{q}q\rangle_\rho = 2\overline{m}\langle\overline{q}q\rangle_0 + \sigma_{\pi N}\rho. \quad (3.1)$$

By taking the ratio to the vacuum condensate and using the GOR relation (2.15) we can cancel the poorly known current mass to get the model-independent result [11]

$$\frac{\langle\overline{q}q\rangle_\rho}{\langle\overline{q}q\rangle_0} = 1 - \frac{\sigma_{\pi N}}{f_\pi^2 m_\pi^2} \rho. \quad (3.2)$$

This was first obtained by Drukarev and Levin in the context of a QCD sum-rule analysis [10], and has also been noted in the NJL and linear sigma models [60, 70, 71]. Assuming that this linear density dependence is valid up to nuclear matter density, $\rho \simeq 0.17 \text{ fm}^{-3}$, and with $\sigma_{\pi N} = 45 \pm 7 \text{ MeV}$ [12], one finds a $\sim 30\%$ reduction in the quark condensate in nuclear matter. Extrapolating to higher densities suggests that chiral symmetry could be completely restored at about three times the density of nuclear matter.

A qualitative picture of the effects of finite baryon density can be obtained from either the linear sigma model or the NJL model. The latter is more often used in calculations of symmetry restoration [86, 87, 60, 11, 71] since it contains a Dirac sea of quarks that can provide a quark condensate. However we have at present no consistent way to describe nuclear matter within this model. Many treatments therefore take the rather drastic step of replacing nuclear matter by a uniform Fermi gas of quarks. This neglects the strong correlations in real nuclear matter that cluster the quarks in threes to form colour-singlet nucleons and then tend to keep those clusters apart. Such correlations lead to partial occupation of many more quark states [88]. Hence a degenerate Fermi gas grossly overestimates the Pauli blocking of quarks in matter. Jaminon *et al* [87] have suggested a hybrid approach, treating the vacuum as a Dirac sea of quarks but using a Fermi sea of nucleons. Although not fully consistent, this approach is probably more realistic than ones based on degenerate quark matter.

In either the linear sigma model or a bosonised NJL model, the energy density for uniform matter can be written

$$\mathcal{E}(\sigma) = U(\sigma) + \frac{\gamma}{(2\pi)^3} \int_0^{k_F} d^3\mathbf{k} \sqrt{\mathbf{k}^2 + M^2}, \quad (3.3)$$

where $U(\sigma)$ is a Mexican hat potential, the fermion mass is $M = g\sigma$, and the degeneracy factor is $\gamma = 4$ for a gas of nucleons and $\gamma = 12$ for one of quarks. These contributions to the energy density in the linear sigma model are shown in Fig. 3.1. For large values of σ or low densities the fermion energy is linear in the baryon density ρ :

$$\mathcal{E}(\sigma) \simeq U(\sigma) + \rho g|\sigma|. \quad (3.4)$$

At low densities this produces a reduction in the vacuum value of σ , and hence in the quark condensate, which is also linear in the density. The consequences of this include reductions in the nucleon mass and the pion decay constant in matter.

As the density increases further the scalar field is pushed to smaller values where the fermions start to become relativistic and so couple less strongly to σ . Finally a density is reached where the maximum at the centre of the Mexican hat is replaced by a minimum. In the case of a quark gas in the absence of explicit symmetry breaking, this

corresponds to a second-order phase transition above which chiral symmetry is restored, the condensate vanishes and fermions are massless. When current masses are included, there is no actual phase transition: the condensate and masses go rapidly but smoothly to small values. For the low sigma masses typical of the NJL model this happens at about three times the normal density of nuclear matter. In contrast, a gas of nucleons remains nonrelativistic to higher densities, and so couples more strongly to the sigma field. The stronger coupling enhances the nonlinear density dependence, leading to an even lower critical density and a transition which tends to be first order.

These typical features are illustrated in Fig. 3.2 for quark and nucleon Fermi gases in the linear sigma model; they can also be seen rather clearly in the work of Jaminon *et al* [87] for the NJL model. In both quark and nucleon cases the quark condensate initially decreases linearly with density. At higher densities this decrease becomes more rapid and, for nucleons, a Lee-Wick phase transition [89] occurs at about half nuclear-matter density. The behaviour of the quark curve may look more plausible, but one should remember that it describes degenerate quark matter, not nuclear matter. Although such models can provide qualitative sketches of chiral symmetry restoration, their details should not be taken seriously. In particular repulsive forces, such as ω exchange, have not been included and use of a light σ meson substantially overestimates the nonlinear density dependence.

At low densities the behaviour of the quark condensate is given by (3.2), independently of whether a gas of nucleons or quarks is used. The model-independent nature of this result was pointed out by Cohen, Furnstahl and Griegel [11], who obtained it using the Feynman-Hellmann theorem. This derivation is worth pursuing because it provides a connection between the quark condensate in matter and the interactions between the nucleons. It can therefore be applied to more realistic models of nuclear matter than those discussed so far. The energy density of nuclear matter can be written in the form

$$\mathcal{E} = \mathcal{E}_0 + M_N \rho + \delta\mathcal{E}, \quad (3.5)$$

where \mathcal{E}_0 is the vacuum energy density (independent of ρ) and $\delta\mathcal{E}$ includes terms of higher order in ρ , coming from nucleon kinetic energies and potentials. The scalar density of quarks can found by differentiating (3.5) with respect to \bar{m} and applying the Feynman-Hellmann theorem (2.17). This gives (3.1) plus higher-order corrections arising from the binding energy in (3.5). Since the binding energy per nucleon is less than 2% of M_N at the density of nuclear matter, these higher-order corrections are expected to be small.

This expectation is borne out by estimates of the density dependence of the quark condensate in several models of nuclear matter. Chanfray and Ericson [84] have looked at pionic effects on the sigma commutator in matter. They find a strong cancellation between Pauli blocking of the pion cloud and pion exchange incorporating tensor correlations. This leaves a small ($< 10\%$) net enhancement of the symmetry restoration over (3.2). Similar enhancements are found from estimates of contributions from heavier-meson exchanges [11, 85]. In all these models the higher-order effects are rather smaller than those in the linear sigma and NJL models mentioned above.

Although the quark condensate in matter can always be calculated directly from a model wave function, it can also be obtained by taking the soft-pion limit of the pion propagator [1, 21]. This is analogous to the soft-pion theorems leading to the GOR relation (2.14) and connecting $\sigma_{\pi N}$ to πN scattering. M. Ericson [90] has suggested that

the condensate can include not just meson-exchange effects but also a “distortion factor,” coming from rescattering of soft pions in the nuclear medium. Such a factor would tend to reduce the amount of chiral symmetry restoration. However, PCAC imposes relations between the various contributions to the scattering amplitude which ensure that only the symmetry-breaking matrix element survives at the soft point. All such contributions of a given order in the density must be included to be consistent with the constraints of PCAC. Also the pion propagator must of course be defined using the PCAC interpolating field (2.10). The sigma commutator per nucleon evaluated in this way should agree with a direct calculation of it as a symmetry-breaking matrix element without reference to any scattering process. The rescattering term [90] does not satisfy these conditions [85].

These features can be illustrated by a calculation of soft-pion scattering to second order in the density ρ in the linear sigma model [85]. To first order in the density, the scattering amplitude per unit volume is just ρ times the free-nucleon result (2.32). Strictly, the leading correction to this arises from the Fermi motion of the nucleons and is of order $\rho^{5/3}$ but at normal nuclear densities this term is negligible. The most important corrections are thus of second order in the density. For simplicity these can be calculated in the static approximation, neglecting the motion of the nucleons. These second-order contributions include the rescattering term [90], which can be represented diagrammatically in Fig. 3.3(a). However one must also include the one-pion-irreducible (OPI) diagrams for pion off two nucleons. The importance of keeping all terms at a given order in the density has long been known in the context of pion-exchange effects on pion-nucleus scattering [91]. Indeed the cancellations between such terms are essential if the pion-exchange contribution to the sigma commutator is to be obtained in the soft-pion limit [84].

Working at tree level the OPI contributions are given by the diagrams of Figs. 3.3(b-d). These are the σ -exchange corrections to the lowest-order result. Other two-body diagrams, for example pion-exchange terms, do not contribute for static nucleons. The soft-pion scattering amplitude including all the diagrams of Fig. 3.3 corresponds to a scalar quark density of

$$2\overline{m}\langle\overline{q}q\rangle_\rho - 2\overline{m}\langle\overline{q}q\rangle_0 = \sigma_{\pi N}\rho + \frac{3}{2}\frac{\sigma_{\pi N}^2}{f_\pi^2 m_\pi^2}\rho^2. \quad (3.6)$$

For comparison, the rescattering diagram, Fig. 3.3(a), gives an order- ρ^2 correction of similar size but with a coefficient of -1 instead of $\frac{3}{2}$. The net effect at this order is thus an enhancement of symmetry restoration in matter, rather than a reaction against it as suggested by rescattering alone. The scalar density (3.6) agrees with a direct evaluation of the matrix element of the symmetry-breaking term in the model (2.28), as it should because the pion field of the linear sigma model is the interpolating field of PCAC.

It is instructive to rederive this result (3.6) using the Feynman-Hellmann theorem since this makes it clear why σ exchange acts to enhance the sigma commutator. The energy density for this simple model of static nucleons interacting via σ -exchange is

$$\mathcal{E} = M_N\rho - \frac{1}{2}\frac{g^2}{m_\sigma^2}\rho^2. \quad (3.7)$$

For fixed λ and ν the relations (2.29, 30) can be used to evaluate the derivatives of the nucleon and sigma masses with respect to m_π^2 . With these, the derivative of (3.7) yields the same result for the change in the quark condensate as (3.6). Sigma exchange is an

attractive interaction whose strength is increased as we switch off the symmetry breaking ($m_\pi \rightarrow 0$). It therefore tends to enhance the sigma commutator in matter.

Taking $\sigma_{\pi N} = 45$ MeV [12] in (3.6) would suggest that σ exchange increases the sigma commutator per nucleon by about 24 MeV at nuclear matter density. Of course, one should also include other components of the NN interaction. For example, ω exchange is a repulsive force whose strength also increases as $m_\pi \rightarrow 0$ [11], tending to counteract the effect of σ exchange. A simple estimate suggests that it will produce a reduction in the sigma commutator of about 10–15 MeV at nuclear matter densities. Short-range correlations between the nucleons will tend to reduce both of these meson-exchange contributions, leaving rather small deviations from the linear density dependence of the quark condensate.

The repulsive short-range correlations between nucleons obviously cut down the contributions of heavy meson exchanges to the quark condensate. In soft-pion scattering it might look as though they could reduce the σ -exchange effects while leaving the rescattering term of Ref. [90] unchanged. However T. Ericson has shown that strong correlations must also affect the rescattering [92]. In a toy model of isolated static nucleons, whose separations are larger than the range of the πN interaction, the propagation of the pions in the empty space between the nucleons can be expressed in terms of on-shell pions only[93]. This remains true even if one even extrapolates the scattering amplitude so that the incident pion momentum is off-shell. The initial scattering in any term of the soft-pion amplitude thus involves one soft and one on-shell pion. If the PCAC field has been used to define the off-shell extrapolation, then the amplitude is zero by the Adler condition [1, 21]. Hence in the limit of extreme correlations, both the rescattering and the OPI diagrams of Fig. 3.3 vanish. This is of course consistent with the vanishing of the σ exchange contribution to $\sigma_{\pi N}$ in this limit.

It has been pointed out [92] that strong correlations between nucleons could have another equally important consequence if changes in the quark condensate were short ranged. Each nucleon would then be an island of symmetry restoration surrounded by normal vacuum and the spatial average of the quark condensate in matter, described by (3.2), would not be a very relevant quantity: the repulsion between nucleons would mean that quarks of a given nucleon would not feel the chiral symmetry restoration produced by its neighbours. However this picture is not realised since, as described in Sec. 2.6, a major contribution to this restoration arises from two-pion exchange between the nucleons [80]. This has a moderately long range, similar to that of the scalar attractive force, even if the “elementary” σ meson has a large mass. Consequently symmetry restoring effects should not be dramatically suppressed by the short-range correlations between nucleons and partial symmetry restoration could have significant effects on nucleons in nuclei. For instance, the symmetry restoring potential experienced by a nucleon is not expected to be substantially reduced from the estimate of -330 MeV in Sec. 2.6.

The estimates of chiral symmetry restoration just discussed are all based on simplified treatments of the NN interaction and nuclear matter. To improve on these we need calculations based on realistic forces with correlations between nucleons obtained consistently from these forces, for example using a relativistic Brueckner-Hartree-Fock approach [28, 94]. The Feynman-Hellmann approach could be applied to estimate the corresponding quark condensate provided one can make reasonable assumptions about the dependence on \overline{m} (or m_π^2) of the masses and couplings of the exchanged mesons. It should also be possible to estimate how much symmetry restoration a nucleon experiences

in matter, taking into account correlations.

A large change in the quark condensate in matter is important since it may lead to comparable modifications of hadron properties. In particular there would be significant effects on meson and baryon masses. The effects on meson masses and decay constants are described in Sec. 3.2. The masses of pions and kaons are discussed in some detail because of suggestions that kaon condensation could occur in dense nuclear matter. Effects on baryon properties have been estimated using a variety of models and these are summarised in Sec. 3.3. Possible observable consequences of these changes in masses and couplings are described later, in Sec. 4. Applications of QCD sum rules to hadrons in matter are illustrated by the calculation of the nucleon scalar and vector self-energies (Sec. 3.4). Possible changes in hadron properties arising from changes in the gluon condensate are discussed in Sec. 3.5.

3.2. Mesons in matter

In a naive quark-model picture, with constituent quark masses generated dynamically, one would expect the masses of non-strange mesons (except for the pions) and baryons to decrease by a similar amount, related to the quark condensate. Since pions would be Goldstone bosons in the chiral limit their masses should remain small, at least until the symmetry-restoring phase transition. When the symmetry is restored each state should become degenerate with a chiral partner of the opposite parity.

Calculations in the NJL model display this expected behaviour for meson masses, although one should not take seriously any results obtained using a Fermi gas of quarks [86, 60]. The Pauli blocking in such a model strongly affects s -wave $q\bar{q}$ states like the vector mesons. Instead of decreasing as the density increases, their masses behave like that of the pion, remaining rather constant at low density and then rising to meet the masses of their axial-vector chiral partners. The rise of the pion mass at high densities is similarly an artefact of the model. Such behaviour is not seen when a gas of nucleons is used [87]: there the masses of the vector, axial-vector and scalar mesons all decrease with density until chiral symmetry is restored.

QCD sum rules have also been used to study the masses of the ρ meson in matter [95, 96]. A basic problem for an predictions from this approach is that the four-quark condensate $\langle(\bar{q}q)^2\rangle$ plays a dominant role in the sum rule. As mentioned in Sec. 2.5, this condensate is not well determined. Even if the factorised ansatz (2.50) is valid in the vacuum, it is not at all clear that this continues to hold in matter. For instance, at finite temperature the contributions from the pion gas do not have this form [98]. The results for the medium dependence of m_ρ are thus crucially dependent on the assumed behaviour of the four quark condensate. If this condensate is taken to decrease strongly with density, as suggested by the factorised ansatz and (3.2), then a strong decrease of m_ρ is found [95, 96]. This is sufficient to overcome the tendency for the mass to increase because of the effect of Δ -hole excitations on the $\pi\pi$ component of the ρ [97, 99].

As long as the chiral symmetry remains hidden, the pion is an approximate Goldstone boson and large changes in its mass are not expected. The linear dependence on the density of the pion mass can be found from the isoscalar πN scattering amplitude at threshold, usually expressed in terms of a scattering length, $a^{(+)} = -0.010m_\pi^{-1}$ [100]. This scattering length is of order \bar{m} , like m_π^2 in free space, but is very small even by

comparison with other quantities of this order. For example it corresponds to a scattering amplitude at threshold that is about five times smaller than at the soft point (2.32). Delorme, Ericson and Ericson [101] (see also [102]) have pointed out that this gives a very weak density dependence of the pion mass in matter. Similarly the NJL model predicts remarkably small changes in the pion mass at nuclear densities [60, 87]. Lutz *et al* [60] have interpreted this in terms of a screening of the scalar interactions, resulting from the finite size of the quark “quasi-particles” in this model. The pion mass thus has an additional protection against modification, beyond that expected from chiral symmetry. In particular there is no evidence for its rapid decrease, leading to s -wave pion condensation, as has been suggested [103].

The behaviour of kaons in matter is less well understood. There have been repeated suggestions that s -wave kaon condensation could occur at a few times normal nuclear densities, which could have important consequences for supernovae and the formation of neutron stars [104, 105].

Early suggestions [106, 107, 26, 108] relied on a scalar attraction between the kaon and nucleon, driven by a large kaon-nucleon sigma commutator. That requires a large strangeness content of the nucleon which, as discussed in Sec. 2.2, is not realistic. Any such attraction is further reduced by the momentum dependence of the KN scattering amplitude, as the pion case [101, 102]. However the idea is not dead because other sources of attraction have been found. In particular the Weinberg-Tomozawa [1] term provides a current-current interaction that is attractive in K^-N scattering [107, 108, 109, 110]. This interaction is often modelled by ρ and ω exchange in phenomenological treatments [111]. In the K^+N case it is repulsive and so there is no tendency for K^+ condensation to occur [110].

Further effects that can promote the formation of a K^- condensate have been pointed out by Brown *et al* [109, 112]. If a Fermi gas of electrons is present, as in neutron star matter, then a condensate can form when the K^- rest energy drops to the electron chemical potential. The symmetry energy of nuclear matter favours the conversion of neutron matter to nuclear matter with a K^- condensate and so lowers the critical density for condensation.

One approach to this problem [108, 109, 112] makes use of effective chiral Lagrangians from ChPT [42, 18]. The most recent calculations start from a Lagrangian that includes terms up to third order in m_π or the small momenta of the chiral expansion [113, 114]. A $\Lambda(1405)$ field is added to the model to account for the rapid energy dependence of K^-p scattering in the vicinity of that resonance. With an attractive force between the $\Lambda(1405)$ and the nucleon, the model can provide sufficient attraction for K^- condensation at about four times nuclear matter density [114]. However the convergence of the chiral expansion seems rather slow [113], as might be expected from the large kaon mass.

Other approaches are based on scattering amplitudes defined in terms of the PCAC interpolating kaon field [110, 102, 115]. These do not find sufficient attraction for kaon condensation. The difference from the ChPT results is not due to the choice of interpolating field: that is purely a matter of convenience and no observable quantity can depend on it. Rather it arises because the two models contain different dynamics at second order in the density [116]. For the approaches to be consistent, they need to include six-point interactions describing kaon scattering from two nucleons. Such terms give rise to irreducible contributions of order ρ^2 to the kaon self-energy in matter, analogous to

those discussed in Sec. 3.1 for the pion case [85]. Moreover there are many such terms of second order in the chiral expansion of the K -nucleus scattering amplitude; current ChPT calculations are thus incomplete at that order.

To decide whether there is sufficient attraction for kaon condensation, it is clearly essential to have empirical input to tie down the density dependence of the K^- interaction with nuclear matter. The scattering lengths, which control the low density behaviour, are of little help: the K^-n scattering length is poorly determined [117], while the K^-p one is complicated by the $\Lambda(1405)$ resonance just below threshold. The K^+N scattering lengths can at least provide some check on the models used. Perhaps the best sources of such information are kaonic atoms. A recent analysis [118] of these using a density-dependent K^- optical potential does find a substantial K^-N attraction [118], although this cannot be reliably extrapolated to the densities of interest for kaon condensation.

As well as modifying mesons' masses, nuclear matter is expected to alter their decay constants. In the present context the pion decay constant is of particular interest since it is directly related to the hidden nature of chiral symmetry in the QCD vacuum, as indicated by its definition (2.8). Any decrease in f_π in matter therefore provides a signal of partial symmetry restoration. Indeed such a reduction is found in both linear sigma [89, 119, 120, 121] and NJL models [86, 60]. By altering the induced pseudoscalar coupling constant in nuclear matrix elements of the axial current, changes in f_π can have observable effects on rates of muon capture [122].

3.3. Nucleons in matter

Partial restoration of chiral symmetry can also produce changes in nucleon properties. Unfortunately we have as yet no consistent model that describes both the quark structure of nucleons and the binding of those nucleons to form nuclei. A hybrid approach is often used where a quark- or soliton-model nucleon is embedded in mean fields taken from a model for nuclear matter. Alternatively QCD sum rules can be applied to a nucleon propagator in matter. Possible observable consequences of the changes in nucleon properties suggested by these models will be discussed in Sec. 4.

Some authors have taken a density-dependent pion decay constant from the NJL model and used this to construct solitons of either a Skyrme [123] or linear sigma model [124]. Christov and Goeke [125] have studied the properties of an NJL soliton embedded in a Fermi gas of quarks. Another approach is based on either a bag [126] or nontopological soliton model [127] where the quarks are coupled to scalar and vector fields. Despite their differences, all of these models yield qualitatively similar results.

The nucleon mass is found to decrease in matter by about 15–20%. Although significant, this is much smaller than the $\sim 40\%$ reduction found in, for example, the σ - ω model [27]. Charge radii increase by 10–20% together with magnetic moments, and so partial symmetry restoration can provide a mechanism for the often-suggested “swelling” of nucleons in matter [128]. The coupling of the nucleon to the axial current g_A decreases slightly ($\sim 5\%$), as a result of the quarks becoming more relativistic as their mass decreases. That also reduces the coupling of scalar fields to the nucleon [126, 127].

The QCD sum rule approach described in the following section can also lead to a decrease of the nucleon mass with density, although this conclusion is sensitive to assumptions about the behaviour of the four-quark condensate. Both this approach and the models mentioned above have been used to study the difference between proton

and neutron masses in matter; this will be discussed further in Sec. 4.3. Other nucleon properties at finite density have not been studied at finite density using QCD sum rules, although Henley *et al* [129] have pointed out that a sum rule for g_A does suggest that this quantity will decrease if chiral symmetry is restored.

3.4. QCD sum rules

QCD sum rules are an alternative to models, which may provide a more direct way to relate changes in nucleon properties to changes in the various condensates in matter. Drukarev and Levin [10] have applied the sum rule approach to the problem of nuclear binding and saturation. However, as pointed out by Cohen, Furnstahl, Griegel *et al* [66] (hereafter denoted by CFG+), the sum rule method is not precise enough to make meaningful predictions for such quantities. Instead that group have looked at the self-energy of a nucleon in matter. Henley and Pasupathy [130] have carried out a similar calculation, expanding both sides of the sum rules to first order in the density.

In matter the Green's function (2.41) for the nucleon interpolating field can be written in terms of three invariant functions:

$$\Pi(q) = \Pi_s(q^2, q \cdot u) + \not{q}\Pi_q(q^2, q \cdot u) + \not{u}\Pi_u(q^2, q \cdot u), \quad (3.8)$$

where u is the four-velocity of the matter. It is convenient to work in the rest frame of the matter where $q \cdot u = q_0$ is just the energy. CFG+ point out that nucleon and antinucleon propagation in matter are not simply related by charge conjugation. The nucleon pole becomes somewhat broadened in matter as a result of coupling to two-particle-one-hole states, for example. Nonetheless it is still narrow on hadronic scales and so can be reasonably well approximated by a single pole. On the other hand the antinucleon can annihilate in matter and becomes a very broad structure. CFG+ therefore suggest working with a spectral representation for the propagator in terms of q_0 , and choosing the weighting function to emphasise the quasi-nucleon pole at positive energy.

The covariant form for the propagator of a nucleon in the presence of scalar and vector potentials leads to the following expressions for the invariant functions in (3.8):

$$\Pi_s(q_0, |\mathbf{q}|) = -\lambda_N^{*2} \frac{M_N^*}{(q_0 - E_q)(q_0 - \overline{E}_q)} + \dots, \quad (3.9a)$$

$$\Pi_q(q_0, |\mathbf{q}|) = -\lambda_N^{*2} \frac{1}{(q_0 - E_q)(q_0 - \overline{E}_q)} + \dots, \quad (3.9b)$$

$$\Pi_u(q_0, |\mathbf{q}|) = +\lambda_N^{*2} \frac{\Sigma_V}{(q_0 - E_q)(q_0 - \overline{E}_q)} + \dots, \quad (3.9c)$$

where the dots denote continuum contributions and polynomial terms. This has positive- and negative-energy poles at

$$E_q = \Sigma_V + \sqrt{\mathbf{q}^2 + M_N^{*2}}, \quad (3.10a)$$

$$\overline{E}_q = \Sigma_V - \sqrt{\mathbf{q}^2 + M_N^{*2}}, \quad (3.10b)$$

where M_N^* is a nucleon's (Dirac) mass in matter and Σ_V is its vector potential. The residue λ_N^{*2} describes the coupling of the interpolating field (2.44) to the quasi-nucleon.

To suppress the antinucleon contribution, CFG+ split each invariant function into pieces that are even and odd in q_0 :

$$\Pi_i(q_0, |\mathbf{q}|) = \Pi_i^E(q_0, |\mathbf{q}|) + q_0 \Pi_i^O(q_0, |\mathbf{q}|). \quad (3.11)$$

They then apply a Borel transform in q_0 at to the combination $\Pi_i^E(q_0, |\mathbf{q}|) - \overline{E}_q \Pi_i^O(q_0, |\mathbf{q}|)$ at fixed $|\mathbf{q}|$. This is equivalent to a weighting function with a factor of $q_0 - \overline{E}_q$ so that the contribution from the negative-energy pole is removed. The Borel transform is similar to that in (2.42, 43) except that $Q^2 = -q_0^2$. This choice ensures that the sum rules reduce to their vacuum forms at zero density.

For large values of the Borel mass M the transform emphasises large negative q_0^2 with fixed $|\mathbf{q}|$. This corresponds to a region of large, space-like four-momenta where the OPE can be applied. That expansion can be done in a similar manner to the vacuum case; details can be found in the work of CFG+ [66]. The main difference from the vacuum OPE is the appearance of new condensates, such as the dimension-three vector quark condensate,

$$\langle q^\dagger q \rangle_\rho = \frac{3}{2} \rho. \quad (3.12)$$

Other condensates, such as $\langle \bar{q} q \rangle$, are replaced by their values in matter.

A simplified version of the sum rules, analogous to (2.48), which illustrates their main features is

$$\lambda_N^{*2} M_N^* \exp[-(E_q^2 - \mathbf{q}^2)/M^2] = -\frac{M^4}{4\pi^2} \langle \bar{q} q \rangle_\rho + \dots, \quad (3.13a)$$

$$\lambda_N^{*2} \exp[-(E_q^2 - \mathbf{q}^2)/M^2] = \frac{M^6}{32\pi^4} + \dots, \quad (3.13b)$$

$$\lambda_N^{*2} \Sigma_V \exp[-(E_q^2 - \mathbf{q}^2)/M^2] = \frac{2M^4}{3\pi^2} \langle q^\dagger q \rangle_\rho + \dots. \quad (3.13c)$$

The dots denote condensates of dimension four or higher and continuum contributions; the full forms are given by CFG+. Taking ratios of these sum rules one gets a modified version of the Ioffe sum rule (2.49) for the nucleon mass,

$$M_N^* = -\frac{8\pi^2}{M^2} \langle \bar{q} q \rangle_\rho, \quad (3.14)$$

and a vector self-energy given by

$$\Sigma_V = \frac{32\pi^2}{M^2} \rho. \quad (3.15)$$

The qualitative features of these sum rules are consistent with relativistic phenomenology [27, 28, 50, 51]. The change in the scalar condensate (3.2) drives a reduction in the nucleon mass and so provides a scalar attraction, while the vector condensate produces a repulsive vector self-energy.

CFG+ have studied the effects of higher condensates omitted from (3.13) and, with one exception, they find their results to be insensitive to them. The exception is the four-quark condensate $\langle (\bar{q} q)^2 \rangle_\rho$ which, as we saw above, plagues the ρ -meson sum rule too. If that condensate is assumed to vary weakly with density the expectations based on the simplified sum rules are fulfilled: in nuclear matter the nucleon mass is

reduced to roughly 60% of its free-space value and there is a vector self-energy of about 300 MeV. On the other hand, if the condensate is given a strong density dependence, as suggested by the factorised ansatz, then the nucleon mass remains almost unchanged in matter. Since the vector self-energy is still large, this gives a quasi-nucleon energy that is substantially larger than M_N . Such a situation seems unrealistic. However it is clear that more work is required to determine the density dependence of the four-quark condensate $\langle(\bar{q}q)^2\rangle_\rho$ before QCD sum rules can yield reliable predictions for mesons and baryons in matter.

3.5. *Scaling?*

Partial restoration of chiral symmetry is expected to decrease hadron masses in matter. Linear sigma [89, 119, 120] and NJL models [86] also show a reduction in the pion decay constant. The results of such models have led to the suggestion [7] that quantities such as the nucleon, σ and vector meson masses and f_π all behave similarly:

$$\frac{M_N^*}{M_N} \simeq \frac{m_\sigma^*}{m_\sigma} \simeq \frac{m_V^*}{m_V} \simeq \frac{f_\pi^*}{f_\pi}, \quad (3.16)$$

where the stars denote values in matter. Brown and Rho [131] (see also [9]) have extended this idea by proposing that there is a single relevant length scale in nuclear matter, essentially f_π^* , and have suggested that this might be a consequence of the broken scale invariance of QCD, which leads to a single dimensioned parameter Λ_{QCD} in the theory (apart from current quark masses).

The QCD scale anomaly [132] can be incorporated in low-energy effective Lagrangians by adding an extra scalar, isoscalar field, the dilaton [133, 134], whose vacuum expectation value provides the only scale. The self-interaction potential for this field, denoted by χ , is taken to be of the form

$$V(\chi) = a\chi^4 + b\chi^4 \ln(\chi/\chi_0). \quad (3.17)$$

The first term provides a scale-invariant classical potential that on its own would give a vanishing vacuum expectation value for χ . It would also leave the dilaton excitations massless, rather like Goldstone bosons. The second term models the quantum effects responsible for the scale anomaly. It explicitly breaks scale invariance, driving the vacuum to a nonzero value of χ and providing a mass for the dilaton excitations. The single dimensioned parameter of the model is χ_0 , which sets the scale of all other dimensioned masses and couplings. From a scaling of all dimensioned quantities, one finds that the χ field can be related to the trace of the stress-energy tensor by

$$-4b\chi^4 = T_\mu^\mu. \quad (3.18)$$

This trace contains all effects that break scale invariance and in QCD it takes the form [132]

$$T_\mu^\mu = -\frac{9\alpha_s}{8\pi} G_{\mu\nu}^a G^{a\mu\nu} + m_u \bar{u}u + m_d \bar{d}d + m_s \bar{s}s. \quad (3.19)$$

In the vacuum this is dominated by the contribution of the gluon condensate $\langle(\alpha_s/\pi)G_{\mu\nu}^a G^{a\mu\nu}\rangle \simeq (360 \pm 20 \text{ MeV})^4$ [62, 64, 65].

The problem with such an approach is that the scale anomaly of QCD is large and so the theory is not approximately scale invariant [135, 136]. This can be seen

from the fact that the lightest scalar glueball, which one might hope to identify with a dilaton, is estimated to lie at around 1.5 GeV [137, 138]. If the dilaton were light enough compared to other states in the same channel, the relation (3.18) could be used to define an interpolating dilaton field by analogy with the pion field of PCAC (2.10). This could then be used to obtain “soft dilaton theorems” describing the consequences of approximate scale invariance which could be embodied in Lagrangians with a dilaton field. In reality there are many other scalar, isoscalar states in the energy range 1–2 GeV and so a single pole is most unlikely to dominate matrix elements of the stress energy tensor. Hence an interpolating dilaton field, introduced as if it were almost a Goldstone boson, is not a useful ingredient in low-energy effective Lagrangians for QCD.

Even if a dilaton field is introduced, it remains almost unchanged in hadronic matter at normal densities. This has been found in many applications of such models [139, 61] (see [136] for a list of further examples): significant changes in the gluon condensate are not produced inside hadrons or normal nuclear matter if realistic values of the glueball mass and gluon condensate are used.

This stiffness of the gluon condensate is another consequence of the lack of scale invariance of QCD. It is clearly shown in the work of Cohen, Furnstahl and Griegel [11], which uses the trace anomaly to relate the change in the gluon condensate to the energy density of hadronic matter. In stable nuclear matter the change in T_μ^μ is simply the energy density of the matter since the pressure vanishes. For normal nuclear matter this gives a change in the gluon condensate of about 150 MeV fm⁻³. This should be compared with the vacuum gluon condensate of 2200 MeV fm⁻³. Even allowing for a factor of two uncertainty in this condensate, its change in nuclear matter is at most a 15% effect. The fourth root of the condensate, which corresponds to the change in the dilaton field or the change of scale, is altered by no more than 4%.

There are only two ways to get large changes in the gluon condensate at normal densities relative to its vacuum value. One is to take a value for the vacuum condensate very much smaller than that deduced from QCD sum rules. That would mean rejecting the rather well tested applications of those sum rules to charmonium [62, 64, 65]. The other is to use a χ field with a much lighter mass than any scalar meson. The gluon condensate would then be very soft and so its response could be large. Universal scaling would arise in such a model, as noted by Kusaka and Weise [140],¹ since the changes in quark condensate generated indirectly via the gluon condensate would be much larger than those produced directly by the scalar density of quarks in matter. However this would require a light dilaton, even though no such particle is observed. It would also be inconsistent with observed nuclear binding energies, since the scale anomaly provides a connection between these and the change in the gluon condensate. Neither of these choices seems acceptable.

The quark and gluon condensates thus behave very differently at finite density and there are at least two scales relevant to nuclear matter, as recognised in [9]. Moreover the stiffness of the gluon condensate means that any universal scaling is very small at normal nuclear densities. The size of the π N sigma commutator and its associated form

¹ The scaling hypothesis leads to hadron masses that vary as the cube root of the quark condensate. Such a relationship has also been found in a version of the NJL model without taking a very large mass for the scalar meson [61]. However in that model the relationship between the masses and the quark condensate is not a consequence of scaling but instead arises from the artificial choice of a model involving four-body rather than two-body forces between the quarks.

factor [12] show that the quark condensate is significantly deformed in the presence of valence quarks. This can occur even if the “elementary” scalar meson is heavy because of its strong mixing with the two pion channel [24]. Any changes in hadron properties in matter are thus likely to be consequences not of scaling but of the partial restoration of chiral symmetry described in Sec. 3.1.

4. Signals of symmetry restoration

Various experimental observations have now been cited as signals for the modification of hadron properties in nuclear matter, consistent with partial restoration of chiral symmetry. Unfortunately almost none of these provides unambiguous evidence since there are “conventional” mechanisms such as short-range correlations, Δ -hole excitations or meson exchange currents which can generate similar density-dependent effects. The important exception is the axial charge, discussed below. A further complication is the fact that effects arising from nucleon structure can be mimicked by Z-graphs in treatments based on point-like Dirac nucleons, as mentioned in Sec. 2.3.

4.1. Axial Charge

The axial charge operator has long been known as a good probe of chiral symmetry in nuclei [141]. Originally interest focussed on the one-pion-exchange contribution, whose form is governed by a soft-pion theorem and which can produce significant enhancement over the one-nucleon piece [142, 143]. More recently, studies of first forbidden β -decays of nuclei in the lead region have indicated enhancements of $\sim 100\%$ in the effective axial charge of a nucleon in matter [144]. Similar enhancements of $\sim 80\%$ have also been found in the tin region [145]. Calculations of the soft-pion term give only about 50% enhancement [143, 146]. Contributions from higher-order terms in ChPT have been found to be small and only weakly dependent on atomic number [147].

Interest has therefore switched to exchanges of heavier mesons, and in particular scalar mesons. Delorme and Towner [148] have pointed out that Z-graphs involving these can produce a further enhancement in relativistic treatments of nuclei. Detailed calculations based on realistic NN interactions have shown that, combined with pion exchange, these effects can explain the axial charges deduced from first forbidden β -decays [149, 150, 145, 151].

Of the heavy-meson contributions, the direct scalar-exchange term is the most important. Other scalar- and vector-exchange Fock terms, although individually large, tend to cancel [149]. Although Z-graphs are responsible for this enhancement when nucleons are treated as point-like Dirac particles, the result is more general than that type of model. The direct scalar exchange corresponds to a reduction in the nucleon mass and this enhances the effective axial charge to

$$g_A^{c*} = \frac{M_N}{M_N^*} g_A. \quad (3.25)$$

Such behaviour has been noted by Kubodera and Rho [155] in the context of scaling. A similar result is found in a soliton model for a nucleon embedded in mean scalar and vector fields [156]. In that model the change in the nucleon mass is smaller than in

approaches based on Dirac nucleons, as mentioned in Sec. 3.3, and the enhancement of the form (3.25) is similarly reduced as a result. However this is compensated by an additional enhancement arising from changes in the nucleon’s structure.

The reaction $pp \rightarrow pp\pi^0$ close to threshold probes similar physics since the one-nucleon piece involves the axial charge of the nucleon. Moreover isospin considerations mean that the soft pion term does not contribute and the kinematics makes this reaction particularly sensitive to heavy meson exchanges. Recent measurements of the total cross section [152] find it to be about five times that expected from the one-nucleon process. Inclusion of a direct scalar exchange term, analogous to the corresponding effect in the axial charge, gives good agreement with the data [153, 154].

The enhancement of the axial charge indicates that there are large scalar fields in nuclei and that the nucleon mass is significantly reduced in matter. Given that partial restoration of chiral symmetry can contribute significantly to such phenomenological scalar fields, this is strong evidence for such symmetry restoration.

4.2. Other signals

Other suggested evidence is less conclusive because there are conventional mechanisms which can produce similar effects. For example, the coupling to the axial current is believed to be quenched in nuclei [157], although the amount of this cannot be determined model-independently [158]. Such quenching occurs in many models for a nucleon in matter discussed in Sec. 3.3 [123, 124, 125, 126, 127], where the quarks become more relativistic as their mass is reduced. However it has long been known that the effective axial coupling is reduced by core polarisation and Δ -hole effects [159, 160]. Calculations of those are not sufficiently accurate to allow one decide whether any intrinsic quenching is also present. The situation with magnetic moments and g -factors is similar [159, 161, 162].

The quenching of the longitudinal response seen in quasi-elastic electron scattering [163, 164] has often been suggested as a signal for a “swelling” of nucleons in matter [128]. Similar effects are also seen in $(e, e'p)$ reactions [165]. The data can be rather nicely explained by a $\sim 15\%$ increase in the proton’s charge radius [166]. In addition, a $\sim 25\%$ increase in its magnetic moment can fit the increased ratio of transverse to longitudinal responses. These changes are comparable to those found in the models mentioned in Sec. 3.3.

A similar increase in the charge radius can also be produced if vector meson masses decrease in matter [167, 168], assuming that a photon couples to a nucleon at least part of the time via a virtual vector meson. Nucleon magnetic moments are also enhanced by this mechanism. A further increase in the magnetic moments arises from the reduction of the effective nucleon mass, if the moments are inversely proportional to that mass. This gives a good description of the ratio of of transverse to longitudinal responses [168]. There is no way to clearly distinguish between an intrinsic “swelling” of nucleons and a decrease in the masses of vector mesons: both pictures lead to similar observable consequences.

Of course we should remember that a significant fraction of the missing longitudinal strength must be due to short-range NN correlations, and that final-state interactions need to be taken into account. Moreover the agreement of the data with y -scaling indicates that one cannot simply rescale the Q^2 dependence of the electric form factor

by more than a 10% change in the charge radius, and that no significant change in the radius for the magnetic form factor is allowed [164]. However the soliton model calculations of the Bochum group [169] suggest that “swelling” is more complicated than just a simple rescaling of the nucleon form factors; that model gives a good description of the longitudinal response although not of the transverse one. As those authors and others [168] note, the transverse response is complicated by the need to include exchange current effects.

Elastic scattering of K^+ mesons provides a good probe of the interior of nuclei since these particles are only weakly absorbed. Measurements of total cross sections for K^+ scattering from ^{12}C and deuterium [170] find a ratio that is significantly larger than expected from the impulse approximation. A much better agreement with the data is obtained if the nucleon radius is increased by 10% [171]. As with (e, e') scattering, a reduction of vector meson masses can also explain the data [172]. Similar discrepancies between the impulse approximation and data on intermediate-energy proton scattering have also been removed by allowing nucleon and vector meson masses to decrease in matter [175]. However Caillon and Labarsouque [173] have noted that decreasing the nucleon as well as meson masses could lead to too large an effect on K^+ scattering. There may also be significant contributions to kaon scattering from pions being exchanged between the nucleons [174].

The EMC effect [176], seen in deep-inelastic lepton scattering from nuclei, shows that there are differences between the momentum distribution of quarks in a nucleus compared with that for a free nucleon. In particular a depletion of valence quarks is seen for momentum fractions in the region $x \sim 0.3\text{--}0.5$. It is now clear that conventional nuclear binding mechanisms cannot explain the whole effect [177]. An increase in the size of the nucleon makes the momentum distribution for the valence quarks more sharply peaked and so reduces the number of high-momentum quarks. Calculations in bag and soliton models [178] give changes in the quark distributions of nucleons in medium which can describe the data quite well.

4.3. Nolen-Schiffer anomaly

The differences between the energies of mirror nuclei [179] have presented a long-standing problem in nuclear physics, often referred to as the Nolen-Schiffer anomaly (NSA). (For a review see Ref. [180].) An intriguing possibility is that changes in the quark structure of nucleons could lead to a reduction of the neutron-proton mass difference in nuclei and so resolve the anomaly. Henley and Krein [181] estimated the effect of partial symmetry restoration on the mass difference using an NJL model for the up- and down-quark masses combined with a nonrelativistic quark model. They found a decrease of the mass difference with density which was sufficient to explain the NSA. Similar results have been obtained by other authors taking different versions of the NJL model [182, 183] or a bag model embedded in mean scalar and vector fields [184]. Hatsuda *et al* [185], and others [182, 186], have applied QCD sum rules to the problem and also found a decrease in the mass difference. However other models give no effect, or even an increase in the neutron-proton mass difference at finite density [187, 188].

There may also be significant contributions to the NSA from charge-symmetry-breaking forces, predominantly arising from ρ - ω mixing [189]. Schäfer *et al* [186] suggest that such effects may be incorporated in the QCD sum rule approach by including isospin

breaking in the vector self-energy of a nucleon in matter. However such explanations are called into question by indications that ρ - ω mixing may vary rapidly as the mesons are taken off-shell [190] and hence this mechanism may contribute much less than expected from the on-shell mixing strength.

There are further complications in the determination of the NSA from the energies of mirror nuclei. Cohen *et al* [191] have shown that any explanation for the NSA based on the local nuclear density leads to a characteristic pattern for nuclei on either side of a closed shell. Such a pattern of shell effects is seen in some extractions of the NSA [192], but not in others [180]. Other caveats concerning attempts to explain the NSA have been pointed out by Auerbach [193]. At present it is thus premature to conclude that the NSA is a signal of medium modifications of nucleons; more consistent treatments of both nucleon and nuclear structure are needed.

4.4. Nuclear forces

Changes in either nucleon structure or meson masses will of course affect the forces between nucleons in nuclei. This is unlikely to alter our nonrelativistic pictures of nuclear structure since phenomenological three-body forces are already included [194]. In fact Hosaka and Toki [195] have shown that density-dependent masses gives G-matrix elements which agree fairly well with empirical ones.

On the other hand, relativistic treatments of nuclei involve strong scalar and vector fields which tend to cancel [27]. Even rather modest changes in meson masses or couplings could produce large changes. For example, a σ mass which decreases with density can prevent nuclear matter from saturating. We may therefore have to look for new mechanisms for saturation, such as a decrease in the σN coupling due to changes in the quark structure of the nucleon [126, 127].

A large reduction in the nucleon mass leads to an enhanced spin-orbit force, one of the successes of the Dirac treatment of nuclei [27]. The changes in the nucleon mass indicated by the models mentioned above are less dramatic. However either a swelling of nucleons or a reduction of vector meson masses can produce the required additional increase in the spin-orbit force.

A decrease in the ρ -meson mass would also affect the tensor force between nucleons. It would increase the strength of ρ -exchange tensor interaction, leading to more cancellation with pion-exchange and hence a reduction of the net isovector tensor force [196]. Measurements of polarisation transfer observable in quasi-elastic proton scattering and (p, n) reactions [197] provide some evidence for such a reduction of the tensor and enhancement of the spin-orbit interactions in nuclei [198].

5. Summary

Chiral symmetry is partially restored in nuclei: the model-independent result (1.1) indicates a $\sim 30\%$ reduction in the average quark condensate in nuclear matter. Corrections of higher order in the density are expected to be small, since they are related to nuclear binding energies. They have been estimated for both pion and heavier meson exchanges. Although more complete calculations based on realistic NN forces and including correlations are needed, these estimates indicate that such corrections are indeed small.

The importance of the pion cloud for the scalar quark density of a single nucleon implies that two-pion exchange forms a major part of the symmetry restoration experienced by a nucleon in matter. This suggests a close relation between symmetry restoration and phenomenological scalar fields in nuclei, which are often used to model the attractive force produced by two-pion exchange. The range of two-pion exchange means that, despite the strong correlations between them, nucleons will experience significant symmetry restoration in nuclear matter. Again, calculations including realistic NN correlations are called for.

Consequences of the partial symmetry restoration include decreases in both meson and nucleon masses. Other effects include a reduction in the pion decay constant. Changes to nucleon properties have been estimated in various models, although one should bear in mind that none of these provides a consistent description of nuclei at the quark level. Qualitatively at least the models are in agreement. As the quark condensate decreases nucleon radii and magnetic moments increase, while their axial-current coupling tends to decrease. These changes are driven directly by the reduction in the quark condensate; any universal scaling of hadron properties, related to the gluon condensate via the scale anomaly of QCD, is negligible at normal nuclear densities.

Although pions and kaons, as approximate Goldstone bosons, are expected to behave rather differently from other hadrons, it has been suggested that attractive forces related to the scalar density could lead to s -wave condensation of these mesons at densities a few times that of normal nuclear matter. For the pions and K^+ this has been ruled out, but for the K^- it remains a possibility. Kaonic atoms do provide evidence for a strong K^- -nucleus attraction. However present models which attempt to extrapolate above nuclear-matter density are incomplete and so no definite conclusions can be drawn.

The effects of symmetry restoration on nucleon properties are consistent with a number of experimental observations, although almost none of these provides an unambiguous signal. In general other, more conventional, mechanisms also contribute and it is hard to disentangle any intrinsic change in nucleon couplings from them. Examples of such signals, which have been widely touted as arising from medium modifications but whose interpretation is still unclear, include the quenched longitudinal response seen in quasi-elastic electron scattering, the enhanced K^+ -nucleus total cross sections and the Nolen-Schiffer anomaly. The one exception is the axial charge, whose enhancement shows that there are strong scalar fields in nuclei.

Modifications of hadron properties in matter could significantly alter the forces between nucleons, in particular strengthening the spin-orbit and weakening the tensor NN interactions in nuclei. Effects consistent with such changes have been seen in polarisation transfer observables in proton scattering and (p, n) reactions.

Further work is needed to improve our theoretical models for the structure of nucleons in nuclei, and to clarify the role of short-range correlations and Δ -hole excitations in the electromagnetic response of nuclei. On the experimental side more information from quasi-elastic electron scattering is needed to help us understand the missing longitudinal strength. Changes in vector meson masses could be investigated directly either by photo- or electroproduction of these mesons from nuclei, or from the decay $\rho \rightarrow e^+e^-$ in warm, dense matter produced in relativistic heavy-ion collisions.

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Figure captions

Fig. 2.1 The Mexican hat potential for the meson fields in the linear sigma model.

Fig. 2.2 Contributions to pion-nucleon scattering in the linear sigma model.

Fig. 2.3 Direct σ exchange diagram.

Fig. 2.4 Diagrams corresponding to two-pion exchange in the linear sigma model: (a) pion loop, (b) pion vertex correction, (c) crossed box, and (d) box.

Fig. 3.1 The energy density of a Fermi gas of nucleons in the linear sigma model (3.3). The potential energy of the meson fields is shown by the solid line, the fermionic energy by the dashed line, and their sum by the dot-dashed line.

Fig. 3.2 The density dependences of the mean sigma field for Fermi gases of quarks (solid line) and nucleons (dashed line). These are for a sigma mass of 600 MeV and include explicit symmetry breaking. The density is expressed in terms of nuclear matter density, $\rho_0 = 0.17 \text{ fm}^{-3}$.

Fig. 3.3 Contributions to pion scattering from two nucleons in the linear sigma model: (a) rescattering, (b-d) one-pion irreducible processes. In (a) each blob denotes the sum of the three diagrams of Fig. 2.2.

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